Should Derivatives be Privileged in Bankruptcy? *

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Abstract

Derivative contracts, swaps, and repos enjoy special status in bankruptcy: they are exempt from the automatic stay and, if collateralized, are effectively senior to virtually all other claims. We propose a simple corporate finance model to assess the effect of these exemptions on a firm's cost of borrowing and incentives to engage in efficient derivative transactions. We show that, while derivatives are value-enhancing risk management tools, effective seniority for derivatives can lead to inefficiencies because it may transfer credit risk to the firm's debtholders, even though this risk could be borne more efficiently in the derivative market. Effective seniority for derivatives is only efficient if it provides sufficient diversification benefits to derivative counterparties that provide hedging services.

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Derivative contracts enjoy special status under U.S. bankruptcy law: Derivative counterparties are exempted from the automatic stay, and—through netting, closeout, and collateralization provisions—they are generally able to immediately collect payment from a defaulted counterparty.[1] Taken together, these special provisions make derivative counterparties *effectively senior* to almost all other claimants in bankruptcy. The costs and benefits of this special treatment are the subject of a recent debate among legal scholars, policymakers, and regulators.[2] Notably, this debate is characterized by considerable disagreement about the costs and benefits of the special bankruptcy treatment of derivatives. This disagreement is also reflected in considerable differences in the bankruptcy treatment of derivatives across different jurisdictions.[3]

In this paper we provide a first formal analysis of the economic consequences of the privileged treatment of derivative contracts in bankruptcy. The fundamental observation underlying our analysis is that (effective) seniority for derivatives does not eliminate default risk—it transfers default risk from derivative counterparties to other claimholders, in particular creditors. The desirability of seniority for derivatives thus depends on whether default risk is more efficiently borne in the derivative market or in the debt market.

To address this question, we extend the standard *limited commitment* framework in corporate finance (Bolton and Scharfstein (1990), Hart and Moore (1994, 1998)) to allow for derivatives as value-enhancing risk-management tools. Within this framework, we are able to characterize conditions under which the current privileged treatment of derivatives is desirable (or undesirable). Our basic model considers a single firm that undertakes a positive net present value (NPV) investment. The investment is financed with debt. Due to operational cash-flow risk, the firm may not have sufficient funds to make the required debt repayment at an intermediate date. Moreover, the firm's limited ability to pledge future cash flows prevents it from rolling over (or renegotiating) its debt following a negative cash-flow shock, such that the firm is forced into default. The firm's limited ability to pledge future cash flows thus constrains its self-insurance capacity and, hence, creates

[1] Similarly, under FDIC receivership there is essentially no stay on derivative contracts. If not transferred to a new counterparty by 5pm EST on the business day after the FDIC has been appointed receiver, derivative, swap, and repo counterparties can close out their positions and take possession of collateral. See, for example, Summe (2010, p.66).

[2] See, e.g., Edwards and Morrison (2005); Bliss and Kaufman (2006); Roe (2011b); Skeel and Jackson (2011); Duffie and Skeel (2012).

[3] For example, under current bank resolution law in the U.K. and Germany, closeout and netting provisions may not always be enforceable (see Hellwig (2011)).

a role for derivatives as hedging tools. Specifically, by allowing transfers of resources from high cash-flow states to low cash-flow states, derivatives can reduce—possibly even eliminate—the incidence of default and inefficient early liquidation. This result is, of course, in line with the existing literature on corporate risk management: When firms face external financing constraints and may be forced into inefficient liquidation, they generally benefit from the ability to hedge cash flow risk (see, e.g., Smith and Stulz, 1985; Froot, Scharfstein, and Stein, 1993).

The main novelty of our analysis is to consider how the bankruptcy treatment of derivatives affects these hedging benefits. The conventional wisdom is that effective seniority for derivatives lower a firm's cost of hedging and should thus be beneficial overall. We show that this argument is (at best) incomplete. While reducing counterparty risk in derivative markets, seniority of derivatives increases the amount of credit risk borne by the firm's creditors. In frictionless financial markets à la Modigliani and Miller, this transfer of risk between different claimants would have no effect on the firm's overall cost of capital. In our incomplete contracting framework, however, this irrelevance logic does not apply, and the priority ranking of debt relative to derivatives matters because it affects (endogenous) contractual frictions in derivative and debt markets.

More specifically, in our model, a net cost of providing hedging services arises endogenously because derivative writers must post collateral to back up their promises as in Biais, Heider, and Hoerova (2012). In essence, when a derivative contract moves against the counterparty that provides the hedge, this counterparty has to post collateral to prevent it from engaging in *risk-shifting* actions that increase *counterparty risk* for the firm. This posting of collateral is costly because it requires giving up other, more productive uses of the counterparty's capital. The priority ranking of derivatives relative to debt affects the net costs of hedging services because it affects the amount of costly collateral that providers of derivatives have to post.

Our analysis suggests that the impact of the priority ordering of derivatives on the overall deadweight costs of hedging depends on the interplay of three main effects. The first effect, which is commonly stressed by practitioners, is that, once the firm has issued its debt, it is (ex post) optimal to hedge default risk with a derivative that is senior to existing debt (or fully backed by collateral from the firm)—when seen in isolation, seniority makes the hedging contract cheaper. Ex ante, however, the firm's creditors anticipate the resulting subordination to derivative positions, which leads to a second, countervailing effect: Creditors demand higher promised repayments to compensate for the higher credit risk they face. The higher required debt payments, in turn, increase the firm's demand for hedging, to the point where the benefits of seniority for derivatives are wiped out by the deadweight costs of higher collateralization requirements for the derivative counterparty that come with the higher hedging demand from firms. Hence, at the firm level, our analysis suggests that it is more efficient if the firm could commit not to grant seniority to its derivative contracts.

The third effect arises when we extend our firm-level analysis to a general equilibrium setting. When derivative counterparties deal with many firms, the diversification benefits to derivative writers from being a senior claimant in bankruptcy can make seniority for derivatives efficient. Specifically, when defaults by firms that use derivatives as hedging tools are imperfectly correlated, payments that senior derivative counterparties receive from defaulted firms can reduce their expected net liabilities in bad states. This, in turn, reduces the amount of collateral that derivative writers are required to post, and can thus reverse the benefits of junior derivatives at the firm level. Our analysis shows that seniority for derivatives is the efficient arrangement when diversification benefits for derivative writers are sufficiently large, which is the case when basis risk on derivative contracts is mostly idiosyncratic. Taken together, our analysis thus suggests that granting seniority to derivative contracts involves a trade-off between (i) the larger net liabilities that derivative writers face vis-à-vis individual firms under seniority for derivatives, and (ii) the diversification benefits that derivative writers can achieve across contracts to different firms when derivative contracts are senior.

We further pursue our analysis by considering how the bankruptcy treatment of derivatives affects the firm's hedging behavior if the firm cannot commit to a hedging policy ex ante and makes its hedging decisions after it has issued debt. We first show that, when derivatives are senior to debt, efficient hedging can become harder to sustain in equilibrium, because the firm may have an incentive to dilute existing debtholders through senior derivative positions it takes ex post. In this situation, it would be more efficient to make derivatives junior to debt, thereby ruling out such inefficient dilution. On the other hand, for some parameter values (e.g., when the firm's continuation value is relatively low), the ability to dilute ex post is *necessary* to sustain hedging: After all, the main beneficiaries from hedging are the firm's creditors, but, when debt is senior, the costs of hedging are mostly borne by shareholders. In such situations, seniority for derivatives can be efficient because it provides a subsidy to encourage efficient hedging that may otherwise not be in the interest of the firm's shareholders.

Finally, we investigate how the seniority treatment of derivatives affects the possibility that the firm may default in high cash flow states, due to losses on its derivative position. Our analysis shows that this outcome is unambiguously more likely when derivatives are senior. First, the combined payment the firm owes on its debt and the derivative is larger when derivatives are senior, making it more likely that the firm does not have sufficient resources to pay. Second, under the current privileged bankruptcy treatment of derivatives, it may be in the counterparty's interest to make an inefficient collateral call that pushes the firm into bankruptcy. If the firm could impose a stay on collateral demands by derivative counterparties, it would be protected against such inefficient collateral calls (or runs on collateral).

To the extent that the favorable bankruptcy treatment of derivatives can lead to inefficiencies, a relevant question is whether firms can contractually 'undo the law' in such cases. For example, firms may want to commit not to collateralize derivative contracts, thus stripping them of their effective seniority. However, debt covenants prohibiting the collateralization of derivatives are likely to be difficult to draft and costly to enforce (see Ayotte and Bolton (2011)). Enforcement constraints are especially severe for financial institutions: While it may be possible to shield physical collateral from derivative counterparties (for example, by granting collateral protection over plant and equipment to secured creditors), it is generally harder to shield unassigned cash from *collateral calls* by derivative counterparties in situations when a financial institution approaches financial distress. By the very nature of their business, financial institutions cannot assign cash as collateral to all depositors and creditors, as this would in effect erase their value added as financial intermediaries. To the extent that firms are unable to contractually undo the effective seniority of derivatives, but can more easily undo a bankruptcy law without any exemptions for derivatives, a change in the bankruptcy code that limits the special treatment of derivatives may be welfare enhancing.

Although a number of legal scholars have informally argued that there may be costs associated with the effective seniority of derivatives (see, e.g., Edwards and Morrison, 2005; Bliss and Kaufman, 2006; Roe, 2011b; Skeel and Jackson, 2011; Duffie and Skeel, 2012),[4] our paper offers the first formal

[4] For example, Edwards and Morrison (2005) argue that one potential adverse consequence of the exemption of the automatic stay is that a firm in financial distress may fall victim to a run for collateral by derivative counterparties. Roe (2011b) argues that fully protected derivative counterparties have no incentive to engage in costly monitoring

ex ante and *ex post* analysis of this issue.[5] In addition to the law literature on the bankruptcy exemption for derivatives and the literature on corporate hedging (see the papers mentioned above), our model is also related to the literature on debt dilution and short-term debt. In particular, in our model the priority ranking of debt and derivative contracts matters because the current bankruptcy code allows firms to dilute their creditors by taking on derivative positions that are effectively senior. This dilution is related to the other classic forms of debt dilution: through risk shifting (e.g., Jensen and Meckling (1976)), via the issuance of additional senior or short-term debt (e.g., Fama and Miller (1972), Diamond (1993a,b), Brunnermeier and Oehmke (2013)), or by granting security interest to some creditors (e.g., Bebchuk and Fried (1996)).

The remainder of the paper is organized as follows. Section 1 briefly summarizes the special status of derivative securities under U.S. bankruptcy law. Section 2 introduces the model. Section 3 analyzes a benchmark case without derivatives. Section 4 discusses the effect of the bankruptcy treatment of derivatives in the case where the derivative has no basis risk. Section 5 extends the analysis to allow for basis risk and presents the main findings of our analysis. Section 6 shows that, in a general equilibrium setting, diversification benefits for counterparties can make seniority for derivatives efficient. Section 7 discusses the effects of the bankruptcy treatment of derivatives on the firm's hedging incentives and on the incidence of strategic default. Section 8 offers some concluding remarks.

1 The Special Status of Derivatives

In this section we briefly summarize the special status of derivatives in bankruptcy and explain why derivatives are often referred so as 'super-senior' claims.[6] Strictly speaking, derivatives are not

of the firm. In addition, commentators have pointed out that under the current rules firms may have an incentive to inefficiently masquerade their debt as derivatives, for example by structuring debt as total return swaps. In this article, we mostly abstract away from ex-post inefficient runs (except in subsection 5.4.2) or inefficient substitution of debt (subject to the automatic stay) for another instrument like debt masquerading as a derivative exempt from the automatic stay.

[5] Recent papers by Antinolfi, Carapella, Kahn, Mills, and Nosal (2012), Acharya, Anshuman, and Viswanathan (2012) and Auh and Sundaresan (2013) also offer an ex ante and ex post analysis of exemptions from the automatic stay, but with a specific focus on repo contracts. Oehmke (2013) provides a model of collateral fire sales that can occur after defaults in the repo markets. Infante (2013) explores the ex ante implications of collateral fire sales.

[6] The discussion in this section is kept intentionally brief and draws mainly on Roe (2011b). For more detail on the legal treatment of derivatives, see also Edwards and Morrison (2005) and Bliss and Kaufman (2006).

senior in the formal legal sense.[7] However, derivatives, swaps and repo counterparties enjoy certain rights that set them apart from regular creditors. While not formally senior, these rights make derivatives *effectively* senior to regular creditors, at least to the extent that they are collateralized.

The most important advantages a derivative, repo or swap counterparty has relative to a regular creditor pertain to closeout, collateralization, netting, and the treatment of eve of bankruptcy payments, eve of bankruptcy collateral calls, and fraudulent conveyances. First, upon default, derivative counterparties have the right to terminate their position with the firm and collect payment by seizing and selling collateral posted to them. This differs from regular creditors who cannot collect payments when the firm defaults, because, unlike derivative counterparties, their claims are subject to the automatic stay. In fact, even if they are collateralized, regular creditors are not allowed to seize and sell collateral upon default, since their collateral, in contrast to the collateral posted to derivative counterparties, is subject to the automatic stay. Hence, to the extent that a derivative counterparty is collateralized at the time of default, collateralization and closeout provisions imply that the derivative counterparty is *de facto* senior to all other claimants.[8]

Second, when closing out their positions with the bankrupt firm, derivative counterparties have stronger netting privileges than regular creditors. Because they can net out offsetting positions, derivative counterparties may be able to prevent making payments to a bankrupt firm that a regular debtor would have to make, thus strengthening the position of derivative counterparties vis-à-vis regular creditors in bankruptcy.[9]

Finally, derivative counterparties have stronger rights regarding eve of bankruptcy payments or fraudulent conveyances. For example, while regular creditors often have to return payments made or collateral posted within 90 days before bankruptcy, derivative counterparties are not subject to

[7] As pointed out by Roe (2011b, p.5), "The Code sets forth priorities in §§ 507 and 726, and those basic priorities are unaffected by derivative status."

[8] If after selling all the posted collateral a derivative counterparty still has a claim on the firm, this remaining claim becomes a regular unsecured claim in Chapter 11. Hence, collateralization is key to the effective seniority of derivative contracts.

[9] The advantages from netting are best illustrated through a simple example. Suppose that a firm has two counterparties, A and B. The firm owes $10 to A. The firm owes $10 to B, and, in another transaction, B owes $5 to the firm. Suppose that when the firm declares bankruptcy there are $10 of assets in the firm. When creditor B cannot net its claims, he has to pay $5 into the firm. The bankruptcy mass is thus $15. A and B have remaining claims of $10 each, such that they equally divide the bankruptcy mass and each receive $7.5. The net payoff to creditor B is $7.5-$5 = $2.5. When creditor B can net his claim, he does not need to make a payment to the firm at the time of default. Rather he now has a net claim of $5 on the bankrupt firm. As before, A has a claim on $10 on the firm. There are now $10 to distribute, such that A receives 2/3*$10 = $6.66 and creditor B receives 1/3*$10 = $3.33. Hence, with netting B receives a net payoff of $3.33, while without netting he only receives $2.5.

those rules. Any collateral posted to a derivative counterparty at the time of a bankruptcy filing is for the derivative counterparty to keep.

Taken together, this special treatment of derivative counterparties puts them in a much stronger position than regular creditors. While they do not have priority in the strict legal sense, their special rights relative to other creditors make derivative counterparties effectively senior, at least to the extent that they are collateralized. In practice, this collateralization is usually ensured via regular marking to market and collateral calls. While for most of the remainder of the paper we will loosely refer to derivatives as being senior to debt, this should be interpreted in the light of the special rights end effective priority of derivative counterparties discussed in this section.

2 Model Setup

2.1 The Firm

We consider a firm that can undertake a two-period investment project. This firm can be interpreted as a non-financial firm undertaking a real investment project or as a financial institution investing in a risky loan or loan portfolio. The investment requires an initial outlay F at date 0 and generates cash flows at dates 1 and 2. At date 1, the project generates high cash flow C_1^H with probability θ, and low cash flow $C_1^L < C_1^H$ with probability $1-\theta$. At date 2, the project generates (expected) cash flow C_2. Following the realization of the first-period cash flow, the project can be liquidated for a liquidation value L. We assume that $0 \leq L < C_2$, implying that early liquidation is inefficient. Unless we explicitly state otherwise, for most of our analysis we normalize the firm's date 1 liquidation value to $L = 0$. After the realization of C_2, the firm is liquidated for a date 2 value of zero.

The firm has no initial funds and finances the project by issuing debt.[10] The debt contract specifies the following terms: (i) a contractual repayment R at date 1;[11] (ii) if the firm makes this contractual payment, it has the right to continue the project and collect the date 2 cash flows; (iii) if the firm fails to make the contractual date 1 payment R, the creditor has the right to discontinue

[10] In the case of a bank, this means that beyond the minimum equity capital requirement, which we normalize to zero, the bank must raise the entire amount needed for the loan in the form of deposits. In what follows, when we interpret the firm as a bank we also take it that the *creditor* is then a bank *depositor*.

[11] In the case of a bank R denotes the gross interest payment on deposits of size F.

the project and liquidate the firm. Liquidation can be interpreted as either an outright liquidation under a Chapter 7 cash auction, or a Chapter 11 reorganization. In the latter interpretation L denotes the expected payment the creditor receives in Chapter 11. Both the firm and the creditor are risk neutral, and the risk-free interest rate is normalized to zero.

The main friction in financing this project is that the firm faces a limited commitment problem similar to Bolton and Scharfstein (1990) and Hart and Moore (1994, 1998). More specifically, we assume that only the minimum date 1 cash flow C_1^L is verifiable, and that all other cash flows can potentially be diverted by the borrower. This means concretely that, even if the high cash flow C_1^H obtains at date 1, the firm can claim to have obtained only the low cash flow and pay out C_1^L instead of R. We also assume that none of the date 2 cash flow can be committed to the lender. Finally, to make financing choices non-trivial, we assume that $C_1^L < F$, such that the project cannot be financed with risk-free debt.

2.2 The Derivative Counterparty

Next, we introduce derivative contracts into the analysis. As with debt contracts, we do this in the simplest possible way. Formally, a derivative contract specifies a payoff that is contingent on the realization of a *verifiable* random variable $Z \in \{Z^H, Z^L\}$. For example, Z could be a financial index or a similar variable that is observable to both contracting parties and verifiable by a court.[12]

A derivative contract of a notional amount X is a promise by the derivative counterparty (described in more detail below) to pay X to the firm if $Z = Z^L$, against a payment x that is payable from the firm to the derivative counterparty when $Z = Z^H$.[13] For simplicity, we assume that Z^L is realized with the same probability as C_1^L, i.e., $\Pr\left(Z = Z^L\right) = 1-\theta$. Hence, a long position in the derivative pays off with the same unconditional probability with which the firm receives the low cash flow C_1^L. The derivative's usefulness in hedging is then determined by the correlation of the derivative payoff with the realization of the low cash flow. We capture this correlation with the parameter γ. Specifically, we assume that Z^L is realized conditional on $C_1 = C_1^L$ with probability

[12] Verifiability of realization of Z and the payment of the amount due under the derivative contract means that, in contrast to cash flows generated by the firm's operations, returns from derivatives positions can be contracted on without commitment or enforceability problems.

[13] The derivative thus has payoffs that are equivalent to a swap contract, one of the most common derivatives used for hedging purposes in practice: It has value zero when entered, and then moves in favor of the firm or the counterparty, depending on the realization of Z.

γ:

$$\Pr\left(Z = Z^L | C_1 = C_1^L\right) = \gamma. \quad (1)$$

Hence, if $\gamma = 1$ the derivative is a perfect hedge for the low cash flow state, since it pays out in exactly the same states in which the firm receives the low cash flow. When $\gamma < 1$, on the other hand, a long position in the derivative only imperfectly hedges the low cash flow state; with probability $(1-\theta)(1-\gamma)$ the derivative does not pay out X even though $C_1 = C_1^L$.[14]

When the firm enters a derivative position, the other side of the contract is what we generically refer to as the derivative counterparty. This counterparty could be a financial institution, an insurance company, or a hedge fund providing hedging services to the firm. Generally the provision of this type of insurance is not free of costs for the derivative counterparty. In particular, when faced with a notional exposure of X, the counterparty may face deadweight costs if it has to post collateral or set aside capital to fulfill capital requirements.

We model these costs as in Biais, Heider, and Hoerova (2012) and consider a derivative counterparty that has assets on its balance sheet, A, that it can optimally invest for a gross expected return of $\Gamma > 1$. When this counterparty enters into a derivative contract it may have to post collateral in a margin account, which only earns the gross risk-free rate (normalized to one). This margin requirement, thus involves a deadweight cost. The reason why the derivative counterparty is required to post collateral is that in the absence of any collateral it may be led to *gamble for resurrection* by inefficiently putting its balance sheet at risk in situations where the odds on the bets it has taken have become less favorable.

More formally, suppose that after entering a derivative contract (X, x)—whereby the counterparty owes X to the firm when $Z = Z_L$ and is promised a payment x by the firm when $Z = Z_H$—the derivative counterparty can take an unobservable action $a \in \{0, 1\}$ which alters the riskiness of its portfolio of assets. When $a = 1$, the return on its assets is deterministic and given by $\Gamma > 1$. When $a = 0$, the return on its assets is risky and equal to Γ only with probability $p < 1$; with probability $(1-p)$ the gross return is equal to zero. The reason why the counterparty may choose action $a = 0$ is that it then obtains a *private benefit* $b > 0$ per unit of assets on the balance sheet.

[14] We have chosen the unconditional payoff probability of the derivative to coincide with the probability that the low cash flow obtains (both are equal to $1-\theta$). This is not necessary for the analysis. We could more generally assume that the derivative pays off with probability $1-p$. Our setup has the convenient feature that when $\gamma = 1$, the derivative is a perfect hedge: it pays if, and only if, the firm's cash flow is low.

We assume that it is efficient to choose $a = 1$:

$$\Gamma > p\Gamma + b. \tag{2}$$

However, the counterparty may prefer to choose $a = 0$ and gamble for resurrection when liabilities build up. In particular, following Biais, Heider, and Hoerova (2012), suppose that before choosing the action a, the counterparty and the firm learn more information about the odds they face on their derivative contract (X, x). For simplicity, suppose they observe a signal $s \in \{s_L, s_H\}$ that is perfectly correlated with Z. When $s = s_H$, incentives are aligned. If the counterparty chooses $a = 1$ its payoff given by

$$A\Gamma + x. \tag{3}$$

Clearly, this is higher than the payoff the counterparty would receive if it chose $a = 0$:

$$A(p\Gamma + b) + x. \tag{4}$$

However, incentives may not be aligned when $s = s_L$. If the counterparty chooses $a = 1$, it now receives

$$A\Gamma - X, \tag{5}$$

which may be lower than the payoff the counterparty receives if it chooses $a = 0$:

$$p(A\Gamma - X) + Ab. \tag{6}$$

This is the case whenever:

$$b \geq \frac{(1-p)(A\Gamma - X)}{A}. \tag{7}$$

Hence, whenever condition (7) holds, the firm must provide incentives to the counterparty to prevent it from choosing action $a = 0$. As Biais, Heider, and Hoerova (2012) have shown, preserving the counterparty's incentives requires that the counterparty post a fraction ζ of its assets as collateral in a margin account, such that the counterparty's *incentive constraint*

$$\zeta A + (1-\zeta)\,A\Gamma - X \geq p(\zeta A + (1-\zeta)A\Gamma - X) + (1-\zeta)Ab, \tag{8}$$

is satisfied. The minimum fraction ζ of assets that needs to be posted as collateral is then given by

$$\zeta = \frac{X - A\mathcal{P}}{A\,(1-\mathcal{P})}, \tag{9}$$

where we defined

$$\mathcal{P} \equiv \Gamma - \frac{b}{1-p}, \tag{10}$$

which can be interpreted as the counterparty's pledgeable income per unit of assets (*unit pledgeable income)*.[15]

Derivatives have economic value in our setting, since the correlation between the derivative payoff and the firm's operational risk can be used to reduce the firm's default risk. In particular, the derivative can be used to decrease the variability of the firm's cash flow at date 1. This effectively raises the verifiable cash flow the firm has available at date 1. From a welfare perspective this is beneficial, because by raising the low date 1 cash flow, the derivative allows the firm to reduce (or even eliminate) the probability of default at date 1. This reduction in the probability of default is socially beneficial, because it reduces the probability that the firm is terminated at date 1. Hence, in the presence of derivatives, the date 2 cash flow C_2 is lost less often. At the same time, the collateral requirement for the counterparty causes a deadweight cost, because per unit of posted collateral the counterparty must forego a net return $\Gamma - 1$. In expectation, the counterparty thus incurs deadweight costs of $(1-\theta)\,(\Gamma-1)\,\zeta$. Using the (9) and defining $\delta \equiv \frac{(1-\theta)(\Gamma-1)}{A(1-\mathcal{P})}$, we can rewrite the deadweight costs as $\delta(X - A\mathcal{P})$, which shows that the derivative counterparty faces a linear deadweight costs for each dollar that its obligation X exceeds pledgeable income on its balance sheet.[16] In a competitive derivatives market, these costs are passed on to the firm. Overall,

[15] If the counterparty were not to post colleteral, it would choose action $a = 0$ when it observes signal s_L and this would result in a loss for the firm, which would only receive the promised payment X with probability $p < 1$. For simplicity, we assume that the derivative counterparty has to make its promises credible by posting collateral. Biais, Heider, and Hoerova (2012) also treat the case of endogenous counterparty risk. In our analysis, the main adjustment from allowing for this case would be that deadweight costs could take the form of either costly collateral or costly endogenous counterparty risk.

[16] When $X \leq A\mathcal{P}$, no collateral needs to be posted, such that no deadweight is incurred. Strictly speaking, the expression for the deadweight costs should thus be $\delta\,(X - A\mathcal{P})^+$. For notational simplicity, we suppress this detail in the remainder of the paper.

derivatives thus increase surplus whenever the gains from reducing date 1 bankruptcy costs outweigh the deadweight cost of using derivatives.[17]

2.3 Seniority treatment of debt and derivative

We model the seniority of derivatives by first considering two extreme cases; first the case where derivatives are senior to debt and then the alternative extreme case in which derivatives are junior. The former situation is one where the required payment to the counterparty x is fully collateralized, and where cash collateral in the amount of x can be seized by the derivative counterparty in the event of a default on debt payments.[18] In the other extreme case when derivatives are junior to debt, the payment to the counterparty x is not collateralized. Moreover, in this case the debt contract also specifies that it is senior to the derivative claim in bankruptcy.

From time to time, we will also consider the more general, intermediate case in which derivatives can be partially collateralized by only assigning a limited cash collateral $\overline{x} \leq x$ to the derivatives counterparty. In this case, only the amount $\overline{x}$ can be seized by the derivatives writer in the event of default. The remaining amount the firm owes to the derivatives counterparty, $x - \overline{x}$, is then treated as a regular debt claim in bankruptcy. For simplicity we will assume that this remainder is junior to the claims of the debtholder.[19]

The treatment of derivatives in bankruptcy affects the payment x promised by the firm when $Z = Z^H$ in the following way: In the event that period 1 cash-flow is C_1^L and that $Z = Z_H$, the firm is unable to meet all its financial obligations—it owes R to its creditor and x to the derivative counterparty, but $R + x > C_1^L$. The priority of debt relative to derivatives therefore affects the size of the payments the derivative counterparty and the lender can expect in this state of the world.

[17] While our discussion above focused on frictions in the derivative market, note that our model treats debt and derivative markets symmetrically: Imposing the same friction that we impose on the derivative counterparty also on the firm's creditor would lead to no change in the model. The firm's creditor never has a net liability to the firm after entering the debt contract, and thus never has to post collateral to preserve incentives.

[18] The cash the firm assigns as collateral to the derivatives margin account is obtained either from retained earnings or from the initial investment by the creditor. Retained earnings can be modeled by assuming that after the firm sinks the set-up cost F at date 0, the project first yields a sure return C_1^L at date 1^-. At that point it is still unknown whether the full period 1 return will be C_1^H or C_1^L; that is, the firm only knows that it will receive an incremental cash flow at date 1 of $\Delta C_1 = C_1^H - C_1^L$ with probability θ, and 0 with probability $(1 - \theta)$. To hedge the risk with respect to this incremental cash flow, the firm can then take a derivative position by pledging cash collateral $x \leq C_1^L$. Alternatively, the cash collateral x can be obtained from the creditor at date 0 by raising a total amount $F + x$ from the creditor. Either way of modeling cash collateral works in our setup.

[19] In practice, such a claim could be classified in the same priority class as debt. We do not explicitly consider this case, since the *pro-rata* allocation of assets to derivative counterparties and debtholders that arises in this case considerably complicates the formal analysis, without yielding any substantive additional economic insights.

We assume that when the derivative is senior to debt, the counterparty is guaranteed to receive x in the event that $Z = Z_H$ (i.e., we assume that $x \leq C_1^L$).[20] If, on the other hand, the derivative is junior to debt, then the counterparty will not receive any payment when $Z = Z_H$ and $C_1 = C_1^L$ since the creditor seizes all assets. This happens with probability $(1-\theta)(1-\gamma)$.

Thus, when the derivative is senior to debt, the payment from the firm in the event that $Z = Z_H$ is the sure payment x, and the break-even constraint for the counterparty is

$$\theta x - (1-\theta) X - (1-\theta)(\Gamma - 1)\zeta = 0. \tag{11}$$

The term $(1-\theta)(\Gamma-1)\zeta$ reflects the expected deadweight cost of collateral that the counterparty is required to post.[21]

In comparison, when the derivative is junior to debt, the counterparty is only paid if the firm receives a high cash flow C_1^H. That is, the payment x^S to the counterparty depends on the realization of basis risk and obtains only with probability $\theta - (1-\theta)(1-\gamma)$. The break-even condition for the derivative counterparty is then given by:

$$[\theta - (1-\theta)(1-\gamma)]\, x^S - (1-\theta) X^S - (1-\theta)(\Gamma-1)\zeta^S = 0, \tag{12}$$

where

$$\zeta^S = \frac{X^S - A\mathcal{P}}{A(1-\mathcal{P})}. \tag{13}$$

As can be seen from (9) and (13), the bankruptcy treatment of derivatives affects required collateral and hence the counterparty's deadweight cost of providing insurance to the firm.

2.4 Timing of moves

Implicit in our description of the model so far is the following assumption on the timing of moves. The firm enters the derivative contract after it has signed the debt contract with the creditor.

[20] The case where $x > C_1^L$ can be treated in an analogous way, but is omitted for brevity.

[21] We follow Biais, Heider, and Hoerova (2012) here by assuming that collateral must not be posted ex ante, and that the contract specifies a collateral requirement only in the event that the derivative contract moves against the counterparty; that is, only when signal s_L is observed. There would be no qualitative change to our analysis if we imposed the collateral requirement up front. In that case, the break-even constraint takes the form $\theta\hat{x} - (1-\theta) X - \zeta(\Gamma-1) = 0$.

Moreover, at the initial contracting stage, the firm and the creditor cannot condition the debt contract on a particular realization of Z. This assumption reflects the idea that at the ex ante contracting stage it may not be known which business risks the firm needs to or can hedge in the future, and what derivative positions will be required to do so. Essentially, this assumption rules out a fully state-contingent contract between the creditor and the firm that bundles financing and hedging at date 0. This assumption is in line with the literature on incomplete contracting.[22,23]

3 Benchmark: No Derivatives

We first describe the equilibrium in the absence of a derivative market. The results from this section provide a benchmark against which we can evaluate the effects of introducing derivative markets in Sections 4 and 5.

In the absence of derivatives, the firm always defaults if the low cash flow C_1^L realizes at date 1. We will refer to this outcome as a *liquidity default*. Because $C_1^L < F$, the low cash flow is not sufficient to repay the face value of debt. Moreover, the date 2 cash flow C_2 is not pledgeable, and since the firm has no other cash it can offer to renegotiate with the creditor, the firm has no other option than to default when C_1^L is realized at date 1. The lender then seizes the cash flow C_1^L and shuts down the firm, collecting the liquidation value of the asset L. Early termination of the project leads to a social loss of $C_2 - L$, the additional cash flow that would have been generated had the firm been allowed to continue its operations.

[22] For a more formal justification of this assumption, assume that there is a continuum of Z-variables that may potentially be used to hedge the firm's business risk, but that at the ex-ante contracting stage it is not yet known which of these potential Z-variables will be the relevant one from a risk management perspective. However, once the firm is in operation and learns more about its business environment it can determine the relevant variable Z. This lack of knowledge on the relevant random variable Z ex ante would effectively prevent the firm from contracting on a particular derivative position or from making the debt contract contingent on the relevant Z-variable. Hence, it is more plausible that the firm chooses its derivative position after signing the initial debt contract. This assumption broadly reflects current market practice: Firms usually choose their derivative exposure for a given amount of debt only ex post. Moreover, in practice relatively few bonds or loans include direct restrictions on future derivative positions taken by the debtor. Nonetheless, we briefly discuss the optimal Z-contingent contract in footnote 31.

[23] One example of a state contingent contract that bundles financing and risk management is when hedging services are provided by the original lender. In contrast to some of the existing literature (see, e.g., Rampini, Sufi, and Viswanathan (2012)), in our framework this bundled contract (when it is feasible) is generally not equivalent to separate hedging and non state-contingent financing contracts. This non-equivalence results from the incentive problem inherent in providing hedging services. Note also that in the presence of this incentive problem, the bundled contract is not necessarily preferable: If the lender's balance sheet is weak, it may be more efficient that the hedge is provided by a counterpart with a strong balance sheet, in order to reduce hedging costs. One advantage of the bundled contract, is that the party providing the bundle internalizes transfers between the financing and hedging portions of the bundle, as also pointed out by Cooper and Mello (1999).

If the high cash flow C_1^H realizes at date 1, the firm has enough cash to service its debt. However, the firm may still choose not to repay its debt. We refer to this choice as a *strategic default*. A strategic default occurs when the firm is better off defaulting on its debt at date 1 than repaying the debt and continuing operations until date 2. In particular, the firm will make the contractual repayment R only if the following incentive constraint is satisfied:

$$C_1^H - R + C_2 \geq C_1^H - C_1^L + S, \tag{14}$$

where S denotes the surplus that the firm can extract in renegotiation after defaulting strategically at date 1. Constraint (14) says that, when deciding whether to repay R, the firm compares the payoff from making the contractual payment and collecting the entire date 2 cash flow C_2 to the payoff from defaulting strategically, pocketing $C_1^H - C_1^L$ and any potential surplus S from renegotiating with the creditor. Repayment of the face value R in the high cash flow state is thus incentive compatible only as long as the face value is not too high:

$$R \leq C_1^L + C_2 - S. \tag{15}$$

The surplus S that the firm can extract in renegotiation with the creditor after a strategic default depends on the specific assumptions made about the possibility of renegotiation and the relative bargaining powers when renegotiation takes place. To keep things simple, we will assume that the creditor can commit not to renegotiate with the debtor and always liquidates the firm after a strategic default. In this case, $S = 0$.[24]

When the incentive constraint (14) is satisfied, the lender's breakeven constraint (given competitive capital markets and our simplifying assumption $L = 0$) is given by

$$\theta R + (1 - \theta) C_1^L = F, \tag{16}$$

[24] This assumption is not crucial for our analysis. We could alternatively assume that renegotiation is possible after a strategic default. For example, one could imagine a scenario in which the firm has full bargaining power in renegotiation. In this case, after a strategic default, the firm would offer $C_1^L + L$ to the creditor, making him just indifferent between liquidating the firm and letting the firm continue. The surplus from renegotiation to the firm would then be given by $S = C_2 - L$ and the project can be financed whenever $F < C_1^L + L$. With slight adjustments, our results on the effects of the priority ranking of derivatives relative to debt carry through in this alternative specification. A sketch of the analysis if under renegotiation is provided in Appendix B of the NBER working paper version of the paper (NPER WP 17599).

which leads to an equilibrium face value of debt of

$$R = \frac{F - (1-\theta) C_1^L}{\theta}. \tag{17}$$

Inserting this expression for the face value into (15) we find that the project can be financed without strategic default occurring in equilibrium as long as

$$F \leq \overline{F} \equiv C_1^L + \theta C_2. \tag{18}$$

In the absence of derivatives, the project cannot be financed if the IC constraint that governs strategic default is violated, since the creditor cannot break even in that case. We summarize the credit market outcome in the absence of derivatives in the following Proposition.

Proposition 1 *In the absence of derivative markets, the firm can finance the project as long as* $F \leq \overline{F} \equiv C_1^L + \theta C_2$*. When the project can attract financing, the face value of debt is given by* $R = \left[F - (1-\theta) C_1^L\right]/\theta$*, and social surplus is equal to expected cash flows minus the setup cost:* $\theta \left(C_1^H + C_2\right) + (1-\theta) C_1^L - F$*.*

Most importantly for the remainder of the paper, Proposition 1 establishes that, in the absence of derivatives, the firm is always shut down after a low cash flow realization at date 1. This early termination results in loss of the date 2 cash flow C_2, which means that the equilibrium is inefficient relative to the first-best (full commitment) outcome. As we will show in the following section, derivatives can reduce this inefficiency by reducing the risk of default at date 1.

4 Financing with Derivatives: No Basis Risk

In this section and in section 5, we consider the firm's problem of optimal hedging when it can commit to selecting the ex ante optimal derivative contract. We consider first the case where the derivative has no basis risk, for simplicity. This corresponds to the situation where $\gamma = 1$. When there is no basis risk the firm can completely eliminate default risk by choosing an appropriate position in the derivative. As we will see, in this benchmark case, the firm always takes the socially optimal hedging position and the priority ordering of the derivative relative to debt is irrelevant. We shall restrict attention for now and the next section 5 to the subset of parameter values for which the no-strategic-default constraint (14) is satisfied. This is the case as long as C_2 is sufficiently large. We will return to this issue in section 7.2, where we examine how the priority ranking of derivatives affects the firm's incentives to default strategically.

When the firm can commit to the derivative position it will choose ex post it will choose the derivative contract that maximizes the overall surplus: Both the creditor and the derivative counterparty just break even, and all remaining surplus is captured by the firm. The firm will thus choose to hedge whenever it is socially optimal to do so and, since the derivative is costly, when hedging is optimal the firm will always take the minimum position in the derivative that is needed to eliminate default.

We can also immediately see that in this case the priority ranking of debt relative to the derivative is irrelevant from an efficiency standpoint. Whenever the firm chooses to hedge, debt becomes risk free and default will never occur. But when there is never any default, the bankruptcy treatment of debt relative to derivatives is irrelevant.

We can see this more formally by comparing the costs and benefits from hedging in either regime. Eliminating default leads to a gain of $(1-\theta)\,C_2$, since now the firm can be kept alive even after the realization of C_1^L at date 1. The net cost of eliminating default is given by the deadweight cost that needs to be incurred in derivative markets. Since the derivative completely eliminates default when there is no basis risk, debt becomes safe, so that $R = F$, irrespective of the priority ranking of debt relative to derivatives. Hence, the deadweight cost of taking the required derivative position $X = F - C_1^L$ is given by

$$(1-\theta)\,\zeta^*\,(\Gamma - 1)\,, \tag{19}$$

where

$$\zeta^* = \frac{F - C_1^L - A\mathcal{P}}{A\,(1-\mathcal{P})}. \tag{20}$$

The firm chooses to hedge whenever the presence of derivatives raises surplus, which is the case when

$$(1-\theta)\,C_2 - (1-\theta)\,(\Gamma - 1)\left(\frac{F - C_1^L - A\mathcal{P}}{A\,(1-\mathcal{P})}\right) > 0, \tag{21}$$

or

$$(1-\theta)\,C_2 - \delta(F - C_1^L - A\mathcal{P}) > 0, \tag{22}$$

where we defined

$$\delta \equiv \frac{(1-\theta)(\Gamma-1)}{A\,(1-\mathcal{P})}. \tag{23}$$

The expression in (22) reveals that the hedging cost is *linear* in $(F - C_1^L - A\mathcal{P})$, the difference in the counterparty's exposure $(F - C_1^L)$ and its total pledgeable income $A\mathcal{P}$. Condition (22) is satisfied whenever the continuation or going-concern value of the firm C_2 is sufficiently large, or when the cost of hedging is sufficiently low (the pledgeable income $\mathcal{P}$ is sufficiently high).

Proposition 2 *When the derivative has no basis risk* ($\gamma = 1$) *and the firm can commit to a derivative position when entering the debt contract:*

1. *The firm chooses the socially optimal derivative position*
2. *The bankruptcy treatment of derivatives is irrelevant*
3. *Hedging with derivatives raises surplus whenever*

$$(1-\theta)\,C_2 - (1-\theta)\,(\Gamma-1)\left(\frac{F - C_1^L - A\mathcal{P}}{A\,(1-\mathcal{P})}\right) > 0. \tag{24}$$

In section 7.1 we consider the case where the firm cannot commit to the derivative position it will take ex-post. In that case, we will see that the firm's private incentives to hedge are suboptimal. Moreover, making derivatives effectively senior opens the door to ex-post debt dilution in the form of speculative short positions in the derivative (rather than long hedging positions). If the firm cannot commit not to enter such speculative derivative positions, then making derivatives junior to debt is efficient because it discourages such ex-post dilution and leads to optimal hedging decisions by the firm for a strictly larger set of parameters.

5 Financing with Derivatives: Basis Risk

Consider next the case where the derivative contract has basis risk ($\gamma < 1$) and suppose again that the no-strategic-default constraint (14) is satisfied.[25] We begin by establishing a preliminary Lemma about collateralization of the derivative position stating that once the face value of debt is set it is always optimal ex post to maximally collateralize the derivative contract when there is basis risk. The reason is that once R is fixed, collateralization of the derivative contract makes hedging cheaper for the firm. Thus, suppose that the firm can choose to only partially collateralize derivatives by only assigning a limited cash collateral $\overline{x} \leq x$ to the derivatives counterparty, such that only the amount $\overline{x}$ can be seized by the derivatives writer in the event of default. The remaining amount the firm owes to the derivatives counterparty, $x - \overline{x}$ is then treated as a regular debt claim in bankruptcy. For simplicity assume that this remainder is junior to the claims of the debtholder.[26] Then, the following lemma obtains.

Lemma 1 *Once financing has been secured and the face value of debt R has been set, it is optimal to fully collateralize the derivative position ex post. This is because, the cost of the derivative $x(\overline{x})$ is decreasing in the level of collateralization:*

$$\frac{\partial x(\overline{x})}{\partial \overline{x}} < 0. \tag{25}$$

Lemma 1 illustrates the conventional wisdom supporting the collateralization and effective seniority of derivatives: Collateralization and seniority for derivatives makes hedging cheaper, which benefits the firm. By this logic, it is often argued, the full collateralization and concomitant seniority of derivative contracts is optimal. Reducing collateralization or making derivative contracts junior to debt is undesirable, as it raises the cost of the derivative to the firm and makes hedging more expensive.

However, as we argue below, changing the level of collateralization and hence effective seniority of derivatives, while holding the face value of outstanding debt constant is not the correct thought

[25] In section 7.2, we examine how the priority ranking of derivatives relative to debt affects the firm's incentives to default strategically.

[26] In practice, such a claim could be classified in the same priority class as debt. We do not explicitly consider this case, since the *pro-rata* allocation of assets to derivative counterparties and debtholders that arises in this case considerably complicates the formal analysis, without yielding any substantive additional economic insights.

experiment, as in the event of default debtholders and derivative counterparties hold claims on the same pool of assets. Accordingly, changing the effective seniority of derivatives must in equilibrium also have an impact on the pricing of the firm's debt. In fact, we will show below that once we allow the firm's terms in the debt market to adjust in response to effective seniority of derivative contracts, the argument for full collateralization and effective seniority for derivatives is reversed. We show this by contrasting the two polar cases of senior derivatives and junior derivatives. These two cases contain essentially all the economic intuition for why, in a partial equilibrium analysis, an arrangement where derivatives are junior is more efficient. One can easily extend the analysis to the intermediate case, in which derivatives can be partially collateralized.[27]

5.1 Seniority of Derivatives over Debt

The structure in which the derivative has seniority over debt most accurately reflects the current special bankruptcy status of derivatives discussed in Section 1. When the derivative is senior, the counterparty is guaranteed the payment x whenever $Z = Z^H$, as long as $x \leq C_1^L$.[28] For the counterparty to break even, then the expected payment received x must equal the expected payments made $X(1-\theta)$ plus the deadweight cost of hedging $(1-\theta)(\Gamma-1)\zeta$, so that:

$$\theta x = (1-\theta)\left[X + (\Gamma - 1)\zeta\right], \tag{26}$$

where

$$\zeta = \frac{X - A\mathcal{P}}{A(1-\mathcal{P})}. \tag{27}$$

Substituting for ζ in (26) and rearranging we obtain the following expression for the counterparty's break-even constraint:

$$\theta x = (1-\theta)X + \delta(X - A\mathcal{P}), \tag{28}$$

[27] To the extent that the incentive to collateralize ex post is undesirable, an important question is whether the firm can commit ex ante not to collateralize its derivative position ex post, for example using covenants. Under current U.S. bankruptcy law this is difficult: If a breach of such a covenant is discovered in bankruptcy, the colleteral has already left the firm and generally cannot be recovered by lenders. Hence, such covenants would require significant monitoring.

[28] When $x > C_1^L$, the counterparty cannot receive more than the whole cash flow C_1^L in the event where $Z = Z^H$ and $C_1 = C_1^L$. In the interest of brevity we focus on the first case, $x \leq C_1^L$, in the main text. The second case is covered in the appendix.

where, as before, we define

$$\delta \equiv \frac{(1-\theta)(\Gamma-1)}{A\,(1-\mathcal{P})}. \tag{29}$$

The face value of debt, R, in turn is determined by the creditor's break-even condition. When derivatives are senior to the creditor and $x \leq C_1^L$, this breakeven condition is given by

$$[\theta+(1-\theta)\,\gamma]\,R+(1-\theta)\,(1-\gamma)\left(C_1^L-x\right)=F. \tag{30}$$

This condition states that the expected payments received by the creditor must equal the initial outlay F. Note that the seniority of the derivative contract becomes relevant in the state when $C_1 = C_1^L$ and $Z = Z^H$, which occurs with probability $(1-\theta)\,(1-\gamma)$. In that case, the derivative counterparty is paid its contractual obligation x before the creditor can receive any payment.

When $\gamma < 1$ the derivative is only a partial hedge, as it sometimes does not pay X when $C_1 = C_1^L$ and sometimes pays X when $C_1 = C_1^H$. Nevertheless, hedging can still be valuable for the firm because it reduces the probability of default. The optimal derivative position under full commitment for the firm is the one that just eliminates default when $C_1 = C_1^L$ and the derivative pays X. Default can then be avoided for any derivative position X, such that

$$X \geq R - C_1^L. \tag{31}$$

Thus, by setting $X = R - C_1^L$ the derivative contract just eliminates default in states when $C_1 = C_1^L$ and $Z = Z^L$ (with probability $(1-\theta)\,\gamma$). Increasing the derivative position beyond this level does not generate any additional surplus; it only increases the deadweight hedging cost and is thus inefficient. Because the derivative is an imperfect hedge, the firm may still default when $C_1 = C_1^L$ and $Z = Z^H$ (with probability $(1-\theta)\,(1-\gamma)$).

Substituting the expression for x from (28) into the creditor's break-even condition (30) and setting $X = R - C_1^L$, we obtain the following expression for the face value of debt:

$$[\theta+(1-\theta)\,\gamma]\,R+(1-\theta)\,(1-\gamma)\left[C_1^L-\frac{(1-\theta)\left(R-C_1^L\right)+\delta(R-C_1^L-A\mathcal{P})}{\theta}\right]=F \tag{32}$$

Solving (32) for R, we then obtain the following characterization of the equilibrium when derivatives are senior to debt.

Proposition 3 ***Senior derivatives.*** *Assume that derivatives are senior and that* $x \leq C_1^L$*. Under full commitment, the optimal derivative position is given by*

$$X = R - C_1^L. \tag{33}$$

This leads to an equilibrium face value of debt

$$R = \frac{\theta F - (1-\theta)(1-\gamma)(1-\delta)C_1^L - \delta(1-\theta)(1-\gamma)A\mathcal{P}}{\theta - (1+\delta)(1-\theta)(1-\gamma)}, \tag{34}$$

and a price of the derivative of

$$x = \frac{(1-\theta+\delta)\left(F - C_1^L\right) - \delta(\theta + (1-\theta)\gamma)A\mathcal{P}}{\theta - (1+\delta)(1-\theta)(1-\gamma)}. \tag{35}$$

To gain intuition about Proposition 3, it is useful to consider the special case in which derivatives provide a perfect hedge against the cash flow risk at date 1 ($\gamma = 1$). In this case, debt becomes risk-free ($R = F$), so that the optimal derivative position is given by $X = F - C_1^L$. When the derivative is not a perfect hedge ($\gamma < 1$), on the other hand, debt remains risky even in the presence of derivatives ($R > F$) and the required derivative position increases to $R - C_1^L > F - C_1^L$.

The social surplus generated in the presence of derivatives depends on how effective derivatives are at hedging the firm's cash flow risks. When the derivative has more basis risk (lower γ), it is less effective as a hedging tool, such that the probability of continuation of the firm at date 1, which is given by $\theta + (1-\theta)\gamma$, is lower. Moreover, a higher basis risk increases the deadweight costs of hedging as the required derivative position, $R - C_1^L$, is larger.

Corollary 1 ***Social surplus.*** *The social surplus when the firm chooses a derivative position of* $X = R - C_1^L$ *is given by*

$$\theta C^H + (1-\theta)C_1^L + [\theta + (1-\theta)\gamma]C_2 - F - (1-\theta)(\Gamma - 1)\zeta. \tag{36}$$

Hedging with derivatives raises social surplus when the gain from the greater likelihood of continu-

ation $(1-\theta)\gamma$ *outweighs the hedging cost:*

$$(1-\theta)\gamma C_2 - (1-\theta)(\Gamma-1)\zeta > 0. \tag{37}$$

Given that the fraction of the counterparty's assets ζ that must be collateralized is decreasing in the counterparty's unit pledgeable income $\mathcal{P}$ (see equation (27)), derivatives are more likely to raise overall surplus when the counterparty has higher unit pledgeable income (i.e., is well-capitalized). Note also that the critical values for $\mathcal{P}$ in (37) and F in (36) depend on the derivative's basis risk γ. In particular, the benefit from hedging $(1-\theta)\gamma C_2$ decreases and the cost of hedging $(1-\theta)(\Gamma-1)\zeta$ increases as basis risk increases (γ decreases).

5.2 Derivatives junior to Debt

We now consider the opposite case, where derivatives are junior to debt. As before, the firm defaults at date 1 when it obtains a low cash flow C_1^L and $Z = Z^H$. This happens with probability $(1-\gamma)(1-\theta)$. When derivatives are junior, in this event the lender now receives the entire cash flow C_1^L, while the counterparty receives nothing. Of course, the greater risk the derivative counterparty is now exposed to is passed on to the firm in the form of a higher cost of insurance x. Indeed, the counterparty's break-even constraint now becomes:

$$x^S[\theta - (1-\theta)(1-\gamma)] = (1-\theta)\left[X^S + (\Gamma-1)\zeta^S\right], \tag{38}$$

(where the superscript S refers to the fact that debt is senior). Note that since the counterparty now only receives the promised payment x^S with probability $\theta - (1-\theta)(1-\gamma)$ rather than with probability θ, it requires a higher promised payment, other things equal. The creditor, on the other hand, now receives the entire cash flow in the default state as the senior claimant, so that his break-even constraint becomes

$$[\theta + (1-\theta)\gamma] R^S + (1-\theta)(1-\gamma) C_1^L = F. \tag{39}$$

With a less risky debt claim, the promised face value of debt required to raise F is lower and is given by:

$$R^S = \frac{F - (1-\theta)(1-\gamma) C_1^L}{\theta + (1-\theta)\gamma}. \tag{40}$$

As before, the optimal derivative position is such that $X^S = R^S - C_1^L$, and default only occurs when $C_1 = C_1^L$ and $Z = Z^H$, with probability $(1-\theta)(1-\gamma)$. Using (38) and (40) we can characterize the equilibrium under junior derivatives as follows.

Proposition 4 ***Junior derivatives.*** *Assume that derivatives are junior. Under full commitment, the optimal derivative position is given by*

$$X^S = R^S - C_1^L. \tag{41}$$

This leads to an equilibrium face value of debt

$$R^S = \frac{F - (1-\theta)(1-\gamma) C_1^L}{\theta + \gamma(1-\theta)}, \tag{42}$$

and price of the derivative of

$$x^S = \frac{(1-\theta+\delta)\left[F - C_1^L\right] - \delta\left[\theta + \gamma(1-\theta)\right] A\mathcal{P}}{\left[\theta - (1-\gamma)(1-\theta)\right]\left[\theta + \gamma(1-\theta)\right]}. \tag{43}$$

As with Proposition 3, we can use the results from Proposition 4 to obtain an expression for the net social surplus when derivatives are junior.

Corollary 2 *With junior derivatives, social surplus is given by*

$$\theta C^H + (1-\theta) C_1^L + \left[\theta + (1-\theta)\gamma\right] C_2 - F - (1-\theta)(\Gamma - 1)\zeta^S, \tag{44}$$

where

$$\zeta^S \equiv \frac{R^S - C_1^L - A\mathcal{P}}{A(1-\mathcal{P})}. \tag{45}$$

When derivatives are junior, the introduction of derivatives raises social surplus relative to the *outcome without derivatives whenever*

$$(1-\theta)\,\gamma C_2 - (1-\theta)(\Gamma - 1)\zeta^S > 0. \tag{46}$$

Again, the fraction of the counterparty's assets ζ^S that must be collateralized is decreasing in the counterparty's unit pledgeable income $\mathcal{P}$, so that condition (46) is more likely to be satisfied when the counterparty is well-capitalized.

From the expressions for R and R^S in propositions 3 and 4 it is immediate to see that $R \geq R^S$, and therefore that $\zeta \geq \zeta^S$. Indeed, from (27) and (45), we see that ζ and ζ^S differ only in the face values of the derivative liabilities for the counterparty $X = R - C_1^L$ and $X^S = R^S - C_1^L$. It follows immediately that the net social surplus under senior debt (and junior derivatives) in Corollary 2 is higher than the net social surplus under junior debt (and senior derivatives) in Corollary 1. This is the key economic observation emerging from our analysis: *When debt is senior, the sum of the equilibrium cost of debt and the hedging contract is lower than when derivatives are senior.* We summarize this result in the following proposition.

Proposition 5 ***Comparing surplus under junior and senior derivatives.*** *Relative to the situation without derivatives, hedging with junior derivatives raises surplus more than hedging with senior derivatives.*

Thus, the received wisdom that the seniority of derivatives is desirable (Lemma 1) reverses once one takes into account the transfer of risk from the derivative counterparty to creditors: Net social surplus is strictly higher under junior derivatives than under senior derivatives, except in two special cases. First, when the derivative is a perfect hedge ($\gamma = 1$) so that the firm never defaults, $R = R^S$, and seniority of the derivative contract is irrelevant. Second, when there is no deadweight hedging cost of hedging ($\zeta = \zeta^S = 0$), seniority is also irrelevant, as a result of a Modigliani-Miller type logic: Risk is transferred from debt markets to derivative markets, but net surplus remains unchanged.

6 A General Equilibrium Analysis

So far, we have analyzed the economic effects of the privileged bankruptcy treatment of derivatives in the context of a single firm and a single counterparty. In this section, we reconsider the effects from a general equilibrium perspective, in which derivative counterparties enter derivative contracts with many firms. The purpose of this analysis is to explore whether the senior treatment of derivatives may be efficient once diversification benefits for derivative counterparties are taken into account. To answer this question, we characterize how counterparties' balance sheets are affected by the bankruptcy treatment of derivatives when they provide hedging services to many firms. We then ask how the change in balance-sheet risk affects the counterparties' deadweight cost of providing insurance.

We consider the effects of a market-wide change in the bankruptcy status of derivatives, by analyzing the equilibrium interaction between a *representative derivative counterparty*, that writes many derivative contracts with many different *representative firms*. Formally, suppose that there is now a continuum of identical firms (with unit mass), each faced with the same set-up cost F, identical cash-flow shocks (C_1^L, C_1^H) in period 1, and each receiving a continuation value C_2 if not liquidated. All of these firms are identically funded with the same debt contract, and hedged with the same derivative contract written with the same representative derivative counterparty, which is subject to a free-entry, zero profit condition. The main difference with the preceding analysis is that now the representative counterparty's balance sheet is composed not only of its initial endowment of assets A, but also of a continuum of (identical) derivative contracts (X, x).

While this is a stylized model of a derivative market—in practice there is likely to be considerable heterogeneity across firms and counterparties—this stylized model has the virtue of simplicity and allows us to focus on the core general equilibrium question: the potential diversification benefits to counterparties from being senior to creditors in bankruptcy. Our analysis shows that the main determinant of the diversification benefits that derivative writers gain from seniority is the correlation structure of the shocks that firms are exposed to. We show this by considering three cases: (i) both firms' cash-flow shocks and derivative bets are independent (i.e., the shocks with respect to C_1 and Z are i.i.d. across firms); (ii) firms are exposed to common cash-flow and derivative shocks. (i.e., the shocks with respect to C_1 and Z are perfectly correlated across firms); (iii) firms' derivative bets are independent, but they are exposed to common cash-flow shocks (i.e., the shocks with respect to C_1 are perfectly correlated, but the Z shocks are i.i.d across firms).

As in the preceding analysis, the key question is how the status of derivatives affects the collateral requirements of the derivative counterparty. The main difference to the previous analysis is that this collateral requirement is now based on the derivative counterparty's net liability from *all* derivative contracts. We show first that, when both the cash-flow risk and basis risk that firms are exposed to are idiosyncratic, then the bankruptcy treatment of derivatives is irrelevant. The reason is that, in this case, irrespective of the bankruptcy treatment of derivative contract, the derivative counterparty can perfectly diversify away all risk, making its balance sheet deterministic. Because of this, the derivative counterparty never has a net liability and, hence, is never required to post collateral. Second, we show that when both cash flow risk and basis risk are systematic, then making derivatives senior does not lead to diversification benefits for the counterparty. Hence, our results from the preceding partial equilibrium analysis carry over and making derivative junior to debt is more efficient. Third, we show that, when basis risk is purely idiosyncratic, the diversification benefits that the derivative counterparty gains from seniority are strong enough to reverse the benefits of debt seniority at the firm level. In this case, once diversification benefits for the counterparty are taken into account, making derivative senior is more efficient.

6.1 No Systematic Risk

Suppose first that neither cash-flow risk nor basis risk are systematic. Then the payoff to the representative derivative counterparty is deterministic. By the law of large numbers, a fraction θ of the counterparty's derivative contract move in favor of the counterparty ($Z = Z^H$), while a fraction $1-\theta$ move out of favor ($Z = Z^L$). When derivatives are senior to debt, the balance sheet of the representative counterparty is given by

$$A\Gamma + \theta x - (1-\theta)\,X, \tag{47}$$

(on the assumption that the counterparty is not required to post collateral, which we verify below). Given that the representative counterparty cannot make strictly positive profits in equilibrium it must then satisfy the following zero-profit condition:

$$A\Gamma + \theta x - (1-\theta)\, X = A\Gamma. \tag{48}$$

This, of course, implies that $\theta x - (1-\theta)\, X = 0$, which means that the representative counterparty never has a net liability. Thus, the incentive constraint (8) for $a = 1$ is always satisfied, such that $\zeta = 0$ under assumption (2).

An analogous argument applies when derivatives are junior to debt. In this case, the balance sheet of the representative counterparty (assuming that no collateral needs to be posted) is given by

$$A\Gamma + [\theta - (1-\theta)(1-\gamma)]\, x^S - (1-\theta)\, X^S, \tag{49}$$

which takes into account that the counterparty receives no payment from firms that receive C_1^L and owe x^S to the counterparty. The zero-profit condition thus becomes

$$A\Gamma + [\theta - (1-\theta)(1-\gamma)]\, x - (1-\theta)\, X = A\Gamma. \tag{50}$$

This breakeven condition implies that $[\theta - (1-\theta)(1-\gamma)]\, x - (1-\theta)\, X = 0$, such that the counterparty never has a net liability. Under assumption (2), the incentive constraint (8) is thus satisfied with $\zeta^S = 0$.

The key insight is that when $\zeta = \zeta^S = 0$, there are no deadweight costs for the representative counterparty, irrespective of the relative priority of debt and derivative contracts. Hence, the status of derivatives in bankruptcy is irrelevant. Of course, as equations 48 and 50 reveal the pricing of the derivative contract differs depending on the bankruptcy regime, but this leaves surplus unchanged.

6.2 Cash Flow Risk and Basis Risk are Systematic

Now consider the opposite extreme case, where both cash-flow risk and basis risk are systematic. In this case, when the representative counterparty observes the signal s_H it learns that all derivatives it has written will move against it, such that a significant liability is added to its balance sheet. To preserve the counterparty's incentives not to gamble for resurrection it now must post significant collateral in a margin account and incur deadweight costs.

When derivatives are senior, the counterparty incurs an aggregate liability of X if all derivatives simultaneously move against it, such that it must post a fraction

$$\zeta = \frac{X - A\mathcal{P}}{A(1-\mathcal{P})} \tag{51}$$

of its assets as collateral whenever signal s_H is observed. As a result, the ex ante zero profit condition for the counterparty is given by

$$\theta x - (1-\theta) X - (1-\theta)(\Gamma - 1)\zeta = 0, \tag{52}$$

where the term $(1-\theta)(\Gamma-1)\zeta$ reflects the expected deadweight cost of collateral that the counterparty is required to post.

When derivatives are junior to debt, when $Z = Z^H$ the counterparty receives the payment x^S only if all firms obtain the high cash flow C_1^H (i.e., payment to the counterparty depends on the realization of the aggregate basis risk). Hence, the counterparty receives x^S only with probability $[\theta - (1-\theta)(1-\gamma)]$. When $Z = Z^L$, the counterparty incurs an aggregate liability of X^S and it must post a fraction

$$\zeta^S = \frac{X^S - A\mathcal{P}}{A(1-\mathcal{P})}, \tag{53}$$

of its assets as collateral. The resulting ex ante zero profit condition is given by

$$[\theta - (1-\theta)(1-\gamma)]\, x^S - (1-\theta) X^S - (1-\theta)(\Gamma-1)\zeta^S = 0. \tag{54}$$

Again, the term $(1-\theta)(\Gamma-1)\zeta^S$ reflects the expected deadweight cost of collateral.

Not surprisingly, in this case the comparison of the junior and senior derivative regimes is analogous to the partial equilibrium analysis in the preceding section. Because $R > R^S$ the required notional derivative position is higher under senior derivatives than under junior derivatives: $X > X^S$. This leads to a higher aggregate net liability for the counterparty, such that the required collateral is higher under senior derivatives than junior derivatives, $\zeta > \zeta^S$. This leads to higher deadweight costs under senior derivatives.[29] Essentially, when cash-flow risk and basis risk are

[29] We can calculate breakeven face values exactly as in the partial equilibrium case above, but omit these here for brevity.

perfectly correlated across firms, seniority for derivatives does not generate any diversification benefits for the derivative counterparty. Hence, the only relevant efficiency consideration is to lower the size of the required derivative positions, which is achieved by making derivatives junior to debt in bankruptcy.

6.3 Systematic Cash-Flow Risk and Idiosyncratic Basis Risk

Finally, consider the intermediate case in which cash flow risk is systematic, but basis risk is idiosyncratic. As we will show, in this case, seniority for derivatives reduces the riskiness of the representative counterparty's balance sheet. With a less risky balance sheet, the counterparty is required to post less collateral, lowering the deadweight cost of insurance. In fact, the diversification effect with respect to the counterparty's balance sheet dominates the detrimental effects from higher required derivative positions at the firm level. Hence, when basis risk is purely idiosyncratic, the regime where derivatives are senior to debt is more efficient.

Consider first the case when derivatives are senior. Idiosyncratic basis risk means that, when all firms receive the low cash flow C_1^L, a fraction γ of firms receives a payoff X from the derivative position. For the remaining (unlucky) fraction $1-\gamma$, the derivative moves the wrong way, such that these firms owe a payment x to the derivative counterparty when C_1^L realizes. This fraction $1-\gamma$ of firms defaults, but, because the counterparty is senior, it receives the full payment x (given the assumption $x \leq C_1^L$). When all firms receive the high cash flow C_1^H, a fraction $1-\frac{(1-\theta)(1-\gamma)}{\theta}$ of firms owe x to the counterparty, while the counterparty is required to pay X to a fraction $\frac{(1-\theta)(1-\gamma)}{\theta}$ of firms.

In the high cash flow state, we can thus write the balance sheet of the derivative counterparty as

$$A\Gamma + \left[1 - \frac{(1-\theta)(1-\gamma)}{\theta}\right] x - \frac{(1-\theta)(1-\gamma)}{\theta} X, \tag{55}$$

assuming that no collateral has to be posted, which we verify below. In the low cash-flow state, the counterparty's balance sheet is given by:

$$\zeta A + (1-\zeta) A\Gamma + (1-\gamma) x - \gamma X, \tag{56}$$

where ζ is the fraction of the counterparty's assets that must be posted as collateral so that the incentive constraint

$$\zeta A + (1-\zeta)\, A\Gamma + (1-\gamma)x - \gamma X \geq p(\zeta A + (1-\zeta)A\Gamma + (1-\gamma)x - \gamma X) + (1-\zeta)Ab \qquad (57)$$

is just satisfied. Solving for ζ in the incentive constraint (57), we find that

$$\zeta = \frac{\gamma X - (1-\gamma)\, x - A\mathcal{P}}{A\,(1-\mathcal{P})}. \qquad (58)$$

Equation (58) immediately illustrates a crucial difference to the previous cases (e.g., equation (51)): Under idiosyncratic basis risk, seniority for derivatives allows the counterparty to offset some of the obligation γX through the payments $(1-\gamma)\, x$ that the senior counterparty receives from defaulted firms, reducing the counterparty's net liability in the low cash flow state. Taken together, the ex ante zero profit condition for the counterparty is now given by:

$$\theta x - (1-\theta)\, X - (1-\theta)\,(\Gamma - 1)\,\zeta = 0. \qquad (59)$$

This breakeven condition immediately implies that the counterparty cannot have a net liability in the high cash flow state, since otherwise it could not break even. This confirms that no collateral has to be posted in the high cash flow state.

Now consider junior derivatives. In the high cash-flow state, no collateral has to be posted and the derivative counterparty's balance sheet is given by

$$A\Gamma + \left[1 - \frac{(1-\theta)\,(1-\gamma)}{\theta}\right] x^S - \frac{(1-\theta)\,(1-\gamma)}{\theta} X^S. \qquad (60)$$

In the low cash-flow state, the balance sheet becomes

$$\zeta^S A + \left(1-\zeta^S\right) A\Gamma - \gamma X^S, \qquad (61)$$

where, from the incentive constraint, we can calculate the collateral requirement as

$$\zeta^S = \frac{\gamma X^S - A\mathcal{P}}{A(1-\mathcal{P})}. \tag{62}$$

The zero profit condition for the counterparty is then:

$$[\theta - (1-\theta)(1-\gamma)]\, x^S - (1-\theta)\, X^S - (1-\theta)(\Gamma - 1)\, \zeta^S = 0. \tag{63}$$

As before, which of the two regimes is more efficient comes down to a comparison of ζ and ζ^S. On the one hand, in the junior derivatives regime, the required derivative position X^S is lower than under senior derivatives because the promised face value of debt R^S is lower. To the extent that the derivative position X^S is smaller, the counterparty has to post less collateral. On the other hand, when derivatives are junior, the counterparty obtains no income from firms that owe payment to the counterparty in the low cash-flow state—the entire cash flow of the defaulted firms goes to the senior creditor. This loss of income increases the required fraction of assets that the counterparty must post as collateral. It is a matter of straightforward algebra to verify that this latter effect dominates, because

$$\gamma X^S \geq \gamma X - (1-\gamma)\, x, \tag{64}$$

which implies that $\zeta^S \geq \zeta$, such that seniority for derivatives is more efficient.[30]

We now summarize the discussion of the three cases above in the following proposition.

Proposition 6 ***General Equilibrium Comparison of Junior and Senior Derivative Regimes.*** *In general equilibrium, the seniority treatment of derivatives is irrelevant when basis risk and cash-flow risk are idiosyncratic. It is socially efficient to have debt senior to derivatives when both cash-flow risk and basis risk are systematic. It is socially efficient to have debt junior to derivatives when cash-flow risk is systematic and basis risk is idiosyncratic.*

Taken together, our analysis thus suggests that granting effective seniority to derivative contracts involves a trade-off between (i) the larger net liabilities that derivative writers face vis-à-vis

[30] It is straightforward to check that when cash-flow risk is idiosyncratic and basis risk is systematic there are also significant netting benefits to be obtained and that the regime where derivatives are senior dominates. We omit this case for brevity.

individual firms under seniority for derivatives and (ii) the diversification benefits that derivative writers can achieve across contracts to different firms when derivative contracts are senior. The main insight of Proposition 6 is that this tradeoff depends on the nature of the underlying risks: When basis risk is systematic, credit risk is more efficiently born in the derivatives market, such that junior derivatives are more efficient. Under idiosyncratic basis risk, on the other hand, diversification benefits for derivative counterparty's are large, such that credit risk is more efficiently born in the lending market, making seniority for derivatives is more efficient.

To illustrate the distinction between idiosyncratic and systematic basis risk, consider the following simple example of firms using derivatives to hedge an aggregate risk. Suppose that, at some point during the recent housing boom, firms had decided to hedge their exposure to the real estate market with derivatives. In this situation, basis risk would have been *systematic* if all (or most) firms had entered derivatives referenced to the same index (e.g., the ABX). If, on the other hand, firms had chosen a number of different ways to hedge this risk (e.g., using a number of different indices), then at least some of the resulting basis risk would have been be *idiosyncratic*.

7 Strategic Hedging and Strategic Default

Our main analysis is cast in a static framework, in which the firm can commit ex-ante to an optimal hedging policy. In reality, it is difficult for firms to make such commitments and it is rare to see covenants in debt contracts restricting the firm's hedging options. When firms cannot commit to an optimal hedging policy they will strategically choose their hedging positions ex post to favor equity-holders possibly at the expense of creditors. The privileged treatment of derivatives in bankruptcy affects firms' incentives to enter into strategic hedging positions. In this section we analyze the incentives created by this privileged treatment by considering in turn two key issues, under-insurance or excess speculation, and inefficient collateral calls.

7.1 Hedging or Speculation?

We begin by considering the special case where there is no basis risk ($\gamma = 1$). Recall that in this case under full commitment it is irrelevant for social efficiency whether derivatives are senior or junior. We now show, however, that when the derivative is chosen ex post by the firm in the best interest of its shareholders then the privileged treatment of derivatives invites inefficient strategic behavior.

No basis risk ($\gamma = 1$). If the firm cannot commit to a derivative position, its private ex-post incentives to hedge are lower than the social incentives. Assume that creditors expect that the firm will indeed hedge. Taking the face value of debt $R = F$ as given, it is in the firm's ex post interest to eliminate credit risk by choosing a derivative position of $X = F - C_1^L$ whenever

$$(1-\theta)\, C_2 - (1-\theta)\left[F - C_1^L\right] - \delta\left[F - C_1^L - A\mathcal{P}\right] > 0. \tag{65}$$

The first term in (65) is the benefit to the firm from being able to continue in the low cash flow state. The second term in (65) is the actuarially fair cost of the derivative. The third term captures the deadweight cost of hedging. Comparing this condition to (22), we see that under no commitment the firm's incentives to hedge are strictly lower than is socially optimal. This is an illustration of the well-known observation that equityholders have suboptimal hedging incentives once debt is in place.

As long as the firm can only take long positions in the derivative, the hedging incentives are independent of the bankruptcy treatment of derivatives. If, on the other hand, we allow the firm to take short positions in the derivative, an additional effect emerges and the bankruptcy treatment starts to matter. In particular, if the derivative contract is senior, the firm is able to dilute the creditor by taking a short position in the derivative. By doing so, the firm transfers resources that would usually accrue to the creditor in the default state into the high cash flow state, in which they accrue to the equityholder. Hence, under seniority for derivatives, a derivative that could function as a perfect hedge may well be deployed as a vehicle for speculation or risk-shifting.

To see this formally, assume that $(1-\theta)\, C_2 - \delta\left(F - C_1^L - A\mathcal{P}\right) > 0$, so that it would be socially optimal for the firm to hedge. Under senior derivatives, we now have to compare the firm's payoff from hedging to the payoff from taking no derivatives position, and also the payoff from taking a short position in the derivative. As it turns out, the firm's incentives are such that it always (weakly) prefers taking a short position in the derivative to taking no position at all. Therefore, the firm will hedge in equilibrium only if the payoffs from hedging exceed the payoffs from speculation by taking a short position. Comparing these payoffs, we see that hedging is now privately optimal if, and only if,

$$(1-\theta) C_2 - (1-\theta) \left[F - C_1^L\right] - \delta \left[F - C_1^L - A\mathcal{P}\right] - \frac{(1-\theta)^2 C_1^L + \delta A\mathcal{P}}{1-\theta+\delta} > 0. \tag{66}$$

The additional term relative to (65) shows that hedging is harder to sustain when short positions in the derivative are possible. In addition, in cases where no position in the derivative is optimal, under senior derivatives the firm now always takes an inefficient short position in the derivative.

Proposition 7 *When the derivative has no basis risk* ($\gamma = 1$) *and the firm cannot commit to a derivative position when entering the debt contract*

1. *The firm's private incentives to hedge are strictly less than the social incentives to hedge.*

2. *When only long positions in the derivative are possible, the bankruptcy treatment of derivatives does not matter for efficiency.*

3. *When the firm can take short 'speculative' positions in the derivative, the bankruptcy treatment of derivatives matters: Under senior derivatives, the firm may choose to take a speculative position in the derivative to dilute its creditors. This is strictly inefficient and restricts the set of parameters for which the efficient hedging position can be sustained.*

Proposition 7 illustrates, in the simplest possible setting, one of the first-order inefficiencies of senior derivatives: Rather than being used as hedging tools, seniority for derivatives may lead firms to channel funds away from creditors, in a form of risk shifting. This is not possible when derivatives are treated as junior to debt.[31]

Basis Risk ($\gamma < 1$). Consider first the firm's ex-post private incentives to take a hedging position X^B when derivatives are senior. If C_2 is large enough that the firm finds it optimal to

[31] This simple setting without basis risk is also useful to illustrate the outcome if at date 0 the firm and creditor were able to write a state-contingent contract based on the realization of Z. If the firm can commit not to take on additional derivative positions ex-post, such a state-contingent contract (which can be viewed as contract that bundles financing and hedging) makes the stand-alone derivative redundant: The optimal state contingent contract would set $R\left(Z^L\right) = C_1^L$ to eliminate default in the low cash flow state and $R\left(Z^H\right) = \left[F - (1-\theta) C_1^L\right]/\theta$ to guarantee breakeven. Since default would never occur, the priority ranking of derivatives relative to debt would be irrelevant. If, on the other hand, the firm cannot commit not to take further derivative positions, the priority ranking of the derivative relative to debt still matters. The reason is that, under no commitment, senior derivatives allow the firm to ex-post undo the state contingent contract agreed to at date 0, leading to the same inefficiency as in Proposition 7.

eliminate default, it would never want to take a derivative position that is smaller than $R - C_1^L$. Under senior derivatives it may, however, have an incentive to take a derivative position that strictly exceeds $R - C_1^L$, which is inefficient given the deadweight cost of hedging. To see this, consider the firm's objective function with respect to hedging after it has already committed to a debt repayment of R, given below. If it is privately optimal for the firm to eliminate the default state, the firm's privately optimal derivative position X^B maximizes the firm's private payoff, subject to the constraint that $X^B \geq R - C_1^L$:

$$\max_{X^B \geq R - C_1^L} \theta \left[C_1^H - R + \frac{1-\theta}{\theta} (1-\gamma) X^B - \left[1 - \frac{1-\theta}{\theta} (1-\gamma) \right] x \left(X^B \right) \right] + (1-\theta) \gamma \left[C_1^L + X^B - R \right] + \left[\theta + (1-\theta) \gamma \right] C_2. \tag{67}$$

where the promised payment to the counterparty $x\left(X^B\right)$ is determined by the protection seller's break-even constraint.

To see why the firm enter an inefficiently large derivative position, it is instructive to look at the firm's marginal payoff from increasing its derivative position beyond the optimal size $X = R - C_1^L$:

$$\underbrace{1-\theta}_{\text{marginal derivative payoff}} - \underbrace{\left[1 - \frac{1-\theta}{\theta}(1-\gamma) \right]}_{\leq 1} \underbrace{\left[1 - \theta + \frac{(1-\theta)(\Gamma-1)}{A\,(1-\mathcal{P})} \right]}_{\text{marginal cost of derivative}} \gtrless 0 \tag{68}$$

The first term is the extra derivative payoff to the firm from increasing its derivative position by one unit beyond X. It is equal to $(1-\theta)$ because an increase in the derivative's notional value generates an additional dollar for the firm with probability $(1-\theta)$. The second term is the share of the marginal cost of an additional unit of the derivative that is borne by the firm. The full marginal cost of an additional unit in notional derivative exposure is given by its actuarially fair marginal cost $(1-\theta)$ plus the marginal increase in the hedging cost $\frac{(1-\theta)(\Gamma-1)}{A(1-\mathcal{P})}$. However, this cost is only borne by the firm in states in which it is the residual claimant. In the default state, the marginal cost of the derivative is paid by the creditor, since the derivative is senior to debt. Thus, the firm does not internalize the full cost of increasing its derivative position beyond X, and therefore may have an incentive to enter a derivative position that is larger than first best.

From (68), we then find that the firm's privately optimal derivative position coincides with the optimal derivative position when the derivative has relatively little basis risk $\gamma \geq \overline{\gamma}$. When the derivative has significant basis risk, $\gamma < \overline{\gamma}$, on the other hand, the firm will enter a derivative position that is too large from a social perspective. This implies that the firm will choose to enter an inefficiently large position in the derivative whenever the derivative's basis risk is sufficiently large. Given the linear nature of our endogenous hedging cost, when the firm chooses to increase the derivative position beyond the efficient level, it will completely expropriate the creditor in the default state, by choosing a position X^B such that $x\left(X^B\right) = C_1^L$. This is summarized in the following proposition.

Proposition 8 ***Senior derivatives may lead to inefficiently large derivative positions.*** *Assume that it is privately optimal for the firm to hedge default risk via the derivative. Then, when the firm cannot commit to a derivative position ex ante, the firm's privately optimal derivative position coincides with the optimal derivative position only if $\gamma \geq \overline{\gamma}$. When $\gamma < \overline{\gamma}$, the firm enters a derivative position that is too large from a social perspective, where*

$$\overline{\gamma} = 1 - \frac{\delta\theta}{(1-\theta)(1-\theta+\delta)}. \tag{69}$$

When $\gamma < \overline{\gamma}$, the firm chooses a position X^B such that $x\left(X^B\right) = C_1^L$, so that

$$X^B_{\gamma<\overline{\gamma}} = \frac{\theta}{1-\theta+\delta} C_1^L. \tag{70}$$

The incentive to take inefficiently large derivative positions disappears when derivatives are junior to debt. To see this, consider the firm's ex-post objective with respect to hedging with junior derivatives. The firm's surplus is unchanged relative to (67), except that the promised payment to the derivative $x(X^B)$ is now determined by (38):

$$x\left(X^B\right) = \frac{(1-\theta)\, X^B + \delta\left(X^B - A\mathcal{P}\right)}{\theta - (1-\theta)\,(1-\gamma)}. \tag{71}$$

Differentiating (67) and (71) with respect to X^B then reveals that under junior derivatives the firm has no incentive to take an excessively large derivative position. Indeed, the marginal payoff from increasing the derivative position beyond $X^S = R^S - C_1^L$ is now given by $-\frac{(1-\theta)(\Gamma-1)}{A(1-\mathcal{P})} < 0$. This is intuitive: Under junior derivatives, the firm bears the full marginal cost of an additional unit of derivative exposure. Since the derivative is priced at actuarially fair terms net of the deadweight hedging cost, the firm cannot gain from increasing its derivative exposure beyond $R^S - C_1^L$.

Proposition 9 ***Under junior derivatives there is no incentive to take excessively large derivative positions.*** *When derivatives are junior, the firm chooses the efficient derivative position whenever it is privately optimal for the firm to hedge.*

In a dynamic context, one implication of our analysis is thus that under the current privileged bankruptcy treatment for derivatives, firms may take derivative positions that are excessively large from a social perspective. This is true even though derivatives are fundamentally value-enhancing in our model as risk management tools. As shown above, this incentive to enter excessively large derivative positions is tightly linked to the basis risk of the derivative contract available for hedging. When the derivative has no basis risk, or when basis risk is sufficiently small, the firm has no incentives to take excessively large positions. When, on the other hand, there is a sufficient amount of basis risk, the firm may have an incentive to take on excessive derivative positions, thereby diluting existing creditors. Rather than being a hedging tool, the derivative can then become a vehicle for speculation.

However, it is important to note that this result is not enough to argue that, in terms of ex post hedging incentive, junior derivatives dominate. To see this, we now allow for the possibility that the firm may not have an incentive to hedge at all ex post (ruled out by assumption above). In this situation, seniority of the derivative contract over debt may have a benefit. As is well known, once debt is in place, the benefit to equityholders from hedging is generally less than the total gain to the firm, because the firm's creditors also stand to gain from the firm's hedge (see, for example, Smith and Stulz (1985)). This means that ex post hedging incentives for equityholders can be inefficiently low.[32] Equivalently, equityholders may want to unwind existing hedges once debt is in place (due to risk shifting considerations á la Jensen and Meckling (1976)).

In this situation, there are advantages to having derivatives senior to debt, as then the costs as well as the benefits of hedging will be shared between holders of equity and debt. In our setting,

[32] This point is related to the beneficial role that ex-post dilution can have in mitigating debt overhang problems (see, e.g., Stulz and Johnson (1985) and Diamond (1993b)).

this is the case when C_2 is relatively low. Specifically, it can be shown that when derivatives are junior to debt it is privately optimal for the firm to hedge if $C_2 > \bar{C}_2$, and when derivatives are senior to debt it is privately optimal for the firm to hedge when $C_2 > \tilde{C}_2$. Depending on parameter values, it is possible that $\tilde{C}_2 < \bar{C}_2$, so that a region exists where the firm only chooses to hedge ex post when derivatives are senior to debt.[33]

7.2 Default due to Derivative Losses and Inefficient Collateral Calls

Up to now we have assumed that the required debt and derivative payments are such that the firm meets all its obligations when it receives the high cash flow C_1^H and is required to make a payment on the derivative. While this helped simplify our analysis, this assumption is not without loss of generality. The reason is that the firm can only make the required payment $R+x$ if it has sufficient resources to do so. Moreover, even if there are sufficient resources in the firm, the firm may have an incentive to default strategically in states where payment is due both on debt and the derivative (up to now we simply assumed that this incentive constrained is satisfied).

We now show that default due to derivative losses in the high state is more likely, the higher the level of collateralization (and thus effective seniority) of derivative contracts. Moreover, we show that this problem is exacerbated when the firm has no way of invoking a stay on inefficient collateral calls that may be privately optimal from the perspective of the derivative counterparty.

7.2.1 Default due to Derivative Losses

Suppose that the firm can choose to only partially collateralize derivatives by only assigning a limited cash collateral $\overline{x} \leq x$ to the derivatives counterparty. Recall that in this case, only the amount $\overline{x}$ can be seized by the counterparty in the event of default. The remaining amount the firm owes to the derivatives counterparty, $x - \overline{x}$ is then treated as a regular debt claim in bankruptcy, and for simplicity we assume that this remainder is junior to the claims of the debtholder.

The reason that default in the high state is more likely when the level of collateralization $\overline{x}$ of the derivative is higher is that a higher level of collateralization of the derivative contract leads to

[33] Note, however, that this situation only arises in a region where hedging is less valuable in the first place (because it occurs for low values of C_2; $C_2 \in (\tilde{C}_2, \bar{C}_2)$). In addition, if the firm has an existing hedge that it entered at the same time that it entered the debt contract, seniority of subsequently entered derivatives may allow the firm to unwind an efficient *existing* hedge that the firm would like to be able to commit to ex ante.

a larger *overall* required payment $R(\overline{x}) + x(\overline{x})$ in states where the derivative moves against the firm. Intuitively, while more collateralization generally decreases the cost of the derivative $x(\overline{x})$, this effect is more than outweighed by the concomitant increase in the face value of debt $R(\overline{x})$, so that the overall payment increases. This makes default more likely because it increases the chance of either fundamental or strategic default in the state when the firm receives the high cash flow, but the derivative moves against the firm. This is summarized in the following Proposition.

Proposition 10 ***Default due to losses on the derivative position.*** *The firm meets its payment obligations when it receives the high cash flow but the derivative moves against the firm as long as:*

$$R(\overline{x}) + x(\overline{x}) \leq \min\left[C_1^H, C_1^L + C_2\right]. \tag{72}$$

The higher the level of collateralization for derivatives, the less likely it is that this condition holds:

$$\frac{\partial R(\overline{x})}{\partial \overline{x}} + \frac{\partial x(\overline{x})}{\partial \overline{x}} = \frac{\delta(1-\gamma)(1-\theta)}{[\theta - (1-\gamma)(1-\theta)][\theta + \gamma(1-\theta)]} > 0 \tag{73}$$

Proposition 10 shows that both fundamental and strategic default are more likely when the derivative is more highly collateralized and thus more senior. This also implies that derivatives can eliminate default in the low cash flow state without causing default in the high cash flow state for a smaller set of parameters when derivatives are more collateralized: The possibility of default due to derivative losses in the high state implies that derivatives can serve as hedging tools only if the ex ante setup cost lies below a cutoff value $F(\overline{x})$. This cutoff value is decreasing in the level the derivative's effective seniority, which means that derivatives can serve as effective hedging tools for a larger set of parameters when they are junior to debt.

Corollary 3 *Derivatives can be used to hedge the low cash flow state without causing default in the high cash flow state as long as*

$$F \leq F(\overline{x}) = K_0 C_1^L + K_1 \min\left[C_1^H, C_1^L + C_2\right] + K_2 A\mathcal{P} - K_3 \overline{x}. \tag{74}$$

where K_0 ,K_1, and K_2 are positive constants, and

$$K_3 = \frac{(1-\gamma)(1-\theta)\delta}{\theta+\gamma(1-\theta)+\delta} \tag{75}$$

Since $K_3 \geq 0$, $F(\overline{x})$ is decreasing in the level of collateralization.

7.2.2 Inefficient Collateral Calls

We now slightly extend the model to show how the exemption of derivatives from the automatic stay under Chapter 11 can lead to inefficient collateral calls by the derivative counterparty. To model collateral calls we introduce into the basic model a working capital demand for the firm, which can also play the role of unassigned cash collateral. Specifically, suppose that the firm requires working capital to generate the date 2 cash flow C_2. Let $y = D - F$ be the amount of working capital in the firm, where D is the amount of funding the firm raises at date 0, and F is the amount it spends on fixed investment. Suppose also for simplicity that the firm can only generate the second period cash flow if there is sufficient working capital in the firm:

$$C_2(y) = \begin{cases} V & \text{if} \quad y \geq \kappa \\ 0 & \text{otherwise} \end{cases}, \tag{76}$$

where $V > 0$ and $\kappa > 0$. Moreover, the working capital used to generate C_2 is spent by the firm before the realization of the date 1 cash flow, so that it is no longer available to make payments to the creditor or derivative counterparty at that point.

Consider briefly the outcome absent derivatives: If V is sufficiently large, it is optimal for the firm to hold sufficient working capital; that is, it is optimal to raise $D = F + \kappa$ at date 0, and to hold working capital $y = \kappa$. The payoff to the firm absent derivatives is then given by

$$\theta \left[C_1^H - R + V\right] \tag{77}$$

where R is determined by the following break-even condition:

$$\theta R + (1-\theta) C_1^L = F + \kappa \tag{78}$$

Now consider the outcome in the presence of derivatives when the firm does not have the protection of a stay on collateral calls by the derivative counterparty. As we will show, this may then give rise to inefficient collateral calls on the firm. In particular, if the derivative moves against the firm, the counterparty to the derivative transaction may find it privately optimal to make a collateral call on the firm's cash in order to ensure full payment on the derivative, even if this reduces overall surplus because it drives the firm into default.

More formally, consider the following time line:

1. The firm writes a debt contract with the lender and borrows an amount $D = F + \kappa$.

2. The firm writes derivative contract (x, X) with the counterparty. This contract involves basis risk γ.

3. The counterparty observes the realization of Z before the realization of the first period cash flow; if $Z = Z^H$ counterparty can initiate a procedure to collect x. If there is no stay the counterparty can immediately make a collateral call on the cash available to the firm κ (and subsequently when first-period cash flow is realized on C_1). In that case the firm would be deprived of its working capital, with the consequence that $C_2 = 0$.

4. If the firm has working capital available it spends it *before* the realization of C_1 and then receives $C_2 = V$ at date 2, provided that the firm is not liquidated before then.

5. First-period cash flow is realized; when cash flow is C_1^H and payment is due on the derivative, the firm chooses whether to repay $R + x$ or not; when cash flow is C_1^H and the firm gets a payment X on the derivative, it must decide whether to repay R, and when cash flow is C_1^L and the firm gets a payment X on the derivative, whether to repay R. When cash flow is C_1^L and the firm must make a payment x the firm is liquidated.

6. If the firm continues to the second period and is able to use its working capital it obtains V.

Given this timeline, the firm is exposed to inefficient collateral calls (effectively a run on working capital) if the firm cannot invoke an automatic stay against collateral calls from a derivative counterparty. To see this, suppose that the firm borrows $F + \kappa$ and takes out a derivative promising to pay $X = R - C_1^L$ when $Z = Z^L$, against a payment $x = [(1 - \theta + \delta) X]/\theta$ when $Z = Z^H$ (assuming that the derivative is senior to debt and that $x \leq C_1^L + \kappa$).

In this case, it is a best response for the derivative counterparty to make a collateral call immediately on the realization of Z^H. If it makes such a collateral call, the firm ends up with insufficient working capital, so that $C_2 = 0$. Because of the collateral call, the firm also chooses to strategically default when C_1^H is realized, because when $C_2 = 0$ running away with $C_1^H - C_1^L$ is strictly more profitable than making the required payment $R + x$ on the debt and the derivative: $C_1^H - C_1^L > C_1^H - R - x$. The derivative counterparty, however, is still able to fully recover its claim because $x \leq C_1^L + \kappa$. Note that, even though it is the collateral call that pushes the firm into default, it is privately optimal for the derivative counterparty to ask for collateral. Should it not make that collateral call, the derivative counterparty would lose access to κ and, in the case that C_1^L realizes, can only hope to receive a maximum amount C_1^L at date 1. Thus, making an immediate collateral call is privately optimal for the counterparty if $C_1^L + \kappa \geq x > C_1^L$.

The collateral call by the counterparty is inefficient because it invariably leads to a loss of C_2 (and strategic default) in a state where absent the collateral call the firm would continue. Moreover, the collateral call is not needed for the derivative counterparty to break even. When V is sufficiently large, the firm's incentive to continue can support a high enough payment by the firm such that both the creditor and derivative counterparty break even in expectation. A stay on collateral calls by the counterparty would prevent this outcome.

8 Conclusion

This paper provides a tractable, transparent model to analyze the ex ante and ex post consequences of granting (effective) seniority to derivative contracts. The special treatment of derivatives in bankruptcy has been carved out with the main objective of providing stability to derivative markets. Over the years, exemptions for derivatives, swaps and repo markets have been gradually extended, with the same stability objective in mind, largely as a result of a concerted push by ISDA to codify its 'master agreements' (see Morgan (2008)). Remarkably, however, up to now there has been essentially no systematic analysis of the likely ex post and ex ante consequences of this special bankruptcy treatment. With the exception of a few law articles (most notably Edwards and Morrison (2005), who point to the potential destabilizing effect of the bankruptcy exemptions in the failures of LTCM and Enron), the general presumption was simply that the effect of the privileged bankruptcy treatment for derivatives was to strengthen derivatives markets, to enhance financial stability, and that the effects on firms' cost of debt would be negligible. In contrast, our analysis suggests that, once ex post and ex ante effects of the bankruptcy treatment of derivatives are taken into account, the overall benefits of the special bankruptcy treatment for derivatives are no longer obvious.

Finally, some of the insights of our analysis may have policy relevance beyond the particular setting discussed in this paper. First, carefully taking into account ex ante consequences is likely similarly important with respect to the *Orderly Liquidation Authority* (OLA) for systemically important financial institution, which was created as part of the Dodd-Frank Act of 2010. Under OLA, all *Qualified Financial Contracts* (QFCs), which include swaps, repos and other derivatives, are transferred into a solvent 'bridge bank,' such that counterparties are fully protected and therefore "prohibited from terminating their contracts and liquidating and netting out their positions" (see page 9 in FDIC (2011)). While this responds to the potential ex-post inefficiencies that can result from the exemption of the automatic stay under Chapter 11 bankruptcy (e.g., large-scale collateral liquidations), the treatment of QFCs under OLA may exacerbate ex ante distortions. For example, this solution may incentivize financial institutions to increasingly rely on QFCs as a source of funding, and thereby substitute away from subordinated debt, which will be at risk of substantial haircuts under OLA. Second, our analysis relates to the current debate on moving derivative contracts to clearinghouses. Similar to the transfer of risk to unsecured creditors highlighted in this paper, moving derivatives to clearinghouses reduces credit risk for those parties that are part of the clearinghouse, but *increases* credit risk for those that remain outside of the clearinghouse (see also Roe (2011a)). These repercussions should be taken into account when designing optimal clearinghouse arrangements and determining which contracts and market participants should be included in the clearinghouse.

9 Appendix

Proof of Lemma 1: The steps needed to calculate the cost of the derivative as a function of the level of collateralization $\overline{x}$ are given below in the section characterizing the equilibrium under partial collateralization. As shown there, holding R fixed and assuming that $\overline{x} \leq C_1^L$, the counterparty's breakeven condition implies that

$$x(\overline{x}) = \frac{(1-\theta)\left[R - C_1^L + \delta\left(R - C_1^L - A\mathcal{P}\right)\right] - (1-\theta)(1-\gamma)\overline{x}}{\theta - (1-\theta)(1-\gamma)}. \tag{79}$$

With R held fixed, this implies

$$\frac{\partial x(\overline{x})}{\partial x} = -\frac{(1-\theta)(1-\gamma)}{\theta - (1-\theta)(1-\gamma)} < 0. \tag{80}$$

Hence, taking the face value of debt as given, the cost of the derivative is decreasing in the level of collateralization of the derivative as long as $\overline{x} \leq C_1^L$. When $\overline{x} > C_1^L$, a further increase in collateralization does not change the payoff of the derivative counterparty, such that in this region the cost of the derivative is unchanged.

Senior Derivatives when $x > C_1^L$: Here we describe the equilibrium under senior derivatives when $x > C_1^L$, which we left out in the main body of the text for space considerations. The main difference to the case discussed in the text is that the breakeven conditions for the derivative counterparty and the creditor change. In particular, when $x > C_1^L$ the derivative counterparty receives the entire cash flow when the firm defaults. The breakeven conditions for the creditor and the counterparty become

$$[\theta + \gamma(1-\theta)]R = F \tag{81}$$

$$[\theta - (1-\theta)(1-\gamma)]x + (1-\gamma)(1-\theta)C_1^L = [1-\theta + (\Gamma - 1)\zeta]X. \tag{82}$$

Inserting $X = R - C_1^L$ and solving (81) and (82) for R and x yields

$$R = \frac{F}{\theta + \gamma(1-\theta)} \tag{83}$$

$$x = \frac{(1-\theta+\delta)\left(F - [\theta + \gamma(1-\theta)]C_1^L\right) - \delta[\theta + \gamma(1-\theta)]A\mathcal{P}}{\theta[\theta + \gamma(1-\theta)]}. \tag{84}$$

Characterization of Equilibrium under Partial Collateralization: This section contains the breakeven conditions used to derive the equilibrium under partial collateralization. Under partial collateralization, the derivative counterparty is senior up to an amount $\overline{x}$. The remainder $x - \overline{x}$ is junior to the creditor's claim. The required derivative position is given by

$$X\left(\overline{x}\right) = R\left(\overline{x}\right) - C_1^L. \tag{85}$$

The creditor's and counterparty's breakeven conditions are given by

$$\begin{aligned} \left[\theta + (1-\theta)\gamma\right] R\left(\overline{x}\right) + (1-\theta)(1-\gamma)\left(C_1^L - \overline{x}\right) &= F & (86) \\ \left[\theta - (1-\theta)(1-\gamma)\right] x\left(\overline{x}\right) + (1-\theta)(1-\gamma)\,\overline{x} &= (1-\theta)\left[R\left(\overline{x}\right) - C_1^L + (\Gamma - 1)\,\zeta\left(\overline{x}\right)\right], & (87) \end{aligned}$$

where

$$\zeta\left(\overline{x}\right) = \frac{R\left(\overline{x}\right) - C_1^L - A\mathcal{P}}{A(1-\mathcal{P})}. \tag{88}$$

Solving (86) and (87) for $R\left(\overline{x}\right)$ and $x\left(\overline{x}\right)$, we obtain

$$\begin{aligned} R\left(\overline{x}\right) &= \frac{F - (1-\theta)(1-\gamma)\left(C_1^L - \overline{x}\right)}{\theta + (1-\theta)\gamma} & (89) \\ x\left(\overline{x}\right) &= \frac{(1-\theta+\delta)\left[F - C_1^L\right] - \delta\left[\theta + \gamma(1-\theta)\right]A\mathcal{P} - (1-\theta)(1-\gamma)\left[\theta - (1-\gamma)(1-\theta) - \delta\right]\overline{x}}{\left[\theta - (1-\gamma)(1-\theta)\right]\left[\theta + \gamma(1-\theta)\right]} & (90) \end{aligned}$$

Proof of Proposition 6: Following substitution of X^S, and $\gamma X + (1-\gamma)x$ for their equilibrium values, we can rewrite $\gamma X^S \leq \gamma X - (1-\gamma)\,x$ as

$$\frac{\theta(1-\gamma)\left[(1-\theta+\gamma\delta)\left(F - C_1^L\right) - \delta\left(\theta + (1-\theta)\gamma\right)A\mathcal{P}\right]}{\left[\theta + (1-\theta)\gamma\right]\left[\theta - (1-\theta)(1-\gamma) + \delta\theta(1-\gamma)\right]} \geq 0 \tag{91}$$

The denominator of this expression is positive. To see this, note that

$$\theta - (1-\theta)(1-\gamma) = \Pr\left[C_1 = C_1^H,\ Z = Z^L\right] \geq 0.$$

Hence, (91) is positive if

$$
\begin{aligned}
(1-\theta+\gamma\delta)\left(F-C_1^L\right) &\geq \delta\left(\theta+(1-\theta)\gamma\right)A\mathcal{P} && (92)\\
\frac{F-C_1^L}{\theta+(1-\theta)\gamma} &\geq \frac{\delta}{1-\theta+\gamma\delta}A\mathcal{P}. && (93)
\end{aligned}
$$

From the creditor's breakeven condition under junior derivatives,

$$\theta+(1-\theta)\gamma R^S+(1-\theta)(1-\gamma)C_1^L=F, \tag{94}$$

we know that $R^S-C_1^L=\frac{F-C_1^L}{\theta+(1-\theta)\gamma}$. Substituting this into (93) and multiplying by γ, we obtain

$$\gamma\left(R^S-C_1^L\right)\geq \underbrace{\frac{\gamma\delta}{1-\theta+\gamma\delta}}_{<1}A\mathcal{P}. \tag{95}$$

As long as there are deadweight costs of hedging under junior derivatives (i.e., $\gamma\left(R^S-C_1^L\right)>A\mathcal{P}$), condition (95) is always satisfied. When $\gamma\left(R^S-C_1^L\right)\leq A\mathcal{P}$, no deadweight costs are incurred in either regime, and thus the bankruptcy ordering is irrelevant.

Proof of Proposition 7: The first two statements in the Proposition follow directly from the discussion in the text. To derive equation (66), we need to compare the payoff to the firm from hedging, which is given by the NPV minus the deadweight cost of hedging,

$$\theta C_1^H+(1-\theta)C_1^L+C_2-F-\delta(F-C_1^L-A\mathcal{P}), \tag{96}$$

to the payoff from entering a speculative short derivative position. (The deviation to a speculative short position is always more profitable for the firm than a deviation to taking no derivative position at all.) If, ex ante, creditors expect the firm to hedge and thus set $R=F$, the payoff to the speculative short derivative position is given by

$$\theta\left(C_1^H+X_{short}-F\right)+\theta C_2, \tag{97}$$

where X_{short} is the derivative position that fully expropriates the creditor in the default state (i.e.,

$x_{short} = C_1^L$). X_{short} can be determined from the counterparty's breakeven condition

$$(1-\theta)\, C_1^L = \theta X_{short} + \frac{\theta}{1-\theta} \delta \left(X_{short} - A\mathcal{P} \right). \tag{98}$$

The firm chooses to hedge when (96) exceeds (97), which leads to equation (66).

Proof of Proposition 10: Assume that the firm receives the high cash flow C_1^H but has to make a payment $x(\overline{x})$ on its derivative position. The firm will meet its total payment obligation $R(\overline{x}) + x(\overline{x})$ under two conditions. First, the cash available to the firm must be sufficient, which is the case whenever

$$C_1^H - \left[R(\overline{x}) + x(\overline{x}) \right] \geq 0. \tag{99}$$

Second, the firm must have no incentive to default strategically. This is the case whenever

$$C_1^H - \left[R(\overline{x}) + x(\overline{x}) \right] + C_2 \geq C_1^H - C_1^L. \tag{100}$$

The left hand side is the payoff from making the contractual payment and continuing, whereas the right hand side is the payoff from declaring default, pocketing $C_1^H - C_1^L$ and letting the creditor and the derivative counterparty split C_1^H. Overall, the firm will thus meet its contractual obligations if

$$R(\overline{x}) + x(\overline{x}) \leq \min \left[C_1^H, C_1^L + C_2 \right]. \tag{101}$$

Equation (73) follows from taking the derivatives of equations (89) and (90) and simplifying.

Proof of Corollary 3: The result follows from substituting (89) and (90) into (72) and simplifying. The constants not given in the main text are

$$K_0 = \frac{(1-\theta)(1-\gamma)\left[\theta - (1-\gamma)(1-\theta)\right] + 1 - \theta + \delta}{\theta + \gamma(1-\theta) + \delta}, \tag{102}$$

$$K_1 = \frac{\left[\theta - (1-\gamma)(1-\theta)\right]\left[\theta + \gamma(1-\theta)\right]}{\theta + \gamma(1-\theta) + \delta}. \tag{103}$$

$$K_2 = \frac{\delta\left[\theta + \gamma(1-\theta)\right]}{\theta + \gamma(1-\theta) + \delta} \tag{104}$$

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